1.5 Constructing Algebraic Expressions

→ How many of you like word problems?
→ How many hate them?
→ One of the hardest things about word problems is translating the words into an algebraic expression. That's what we're practicing today.

Ex a) the sum of 7 and 2 — $7+2$

b) The product of 3 and 6 — $3 \cdot 6$

c) 17 decreased by 4 — $17-4$

d) The quotient of 6 and 2 — $6 \div 2$ or $\frac{6}{2}$

→ Sometimes we need variables

Ex a) the difference between 3 and a number

number: x

$3-x$ or $x-3$ ← ambiguous

b) two fifths of a number

number: x

$\frac{2}{5}x$

→ Harder ones

Ex a) 6 more than twice a number

number: x

$6+2x$

b) 3 less than the quotient of a number and 6.

$\frac{x}{6}-3$

[illegible]

c) 5 plus the square of a number $5 + x^2$

d) The quotient of 3 and twice the sum of 2 and a number $\frac{3}{2(2+x)}$

→ More Practical

Ex a) You have x quarters. How many dollars? $\frac{x}{4}$

→ since 1 dollar is equal to 4 quarters, this is like multiplying by 1.

Unit conversion: $x \text{ quarters} \cdot \frac{1 \text{ dollar}}{4 \text{ quarters}} = \frac{x}{4} \text{ dollars}$

b) x dimes and y nickels. How many dollars?

$$x \text{ dimes} \cdot \frac{1 \text{ dollar}}{10 \text{ dimes}} + y \text{ nickels} \cdot \frac{1 \text{ dollar}}{20 \text{ nickels}} = \frac{x}{10} + \frac{y}{20} \text{ dollars}$$

How many cents?

$$x \text{ dimes} \cdot \frac{10 \text{ cents}}{\text{dime}} + y \text{ nickels} \cdot \frac{5 \text{ cents}}{\text{nickel}} = 10x + 5y \text{ cents}$$

→ you can use units to help set up a problem

Ex a) You drive 400 miles in 4 hours (at a constant speed) how fast did you drive?

$$\frac{400 \text{ miles}}{4 \text{ hours}} = 100 \text{ miles per hour} \rightarrow \text{way too fast!}$$

→ in general distance = (rate)(time)

b) You go to jail & your car is impounded at $60 a day for 4 days. How much do you pay?

$$\frac{\$60}{\text{day}} \cdot 4 \text{ days} = \$240$$

→ Percents

→ when you use a percent in a problem, convert it to fractional or decimal form.

Ex

		fraction	decimal
a)	4%	$4/100$	0.04
b)	12%	$12/100$	0.12
c)	89%	$89/100$	0.89
d)	136.5%	$\frac{136.5}{100}$	1.365

→ this is only useful if you have a calculator

Ex a) 25% of a number $\frac{25}{100}x = \frac{1}{4}x$ or $0.25x$

b) 16% of the sum of 3 and a number $\frac{16}{100}(3+x) = \frac{4}{25}(3+x)$

c) You drink x liters of 75% antifreeze. How many liters do you drink?

- $\frac{75}{100} x^{\text{liters}} = \frac{3}{4} x$ liters

→ 75% antifreeze means that for every 100 liters of the fluid, 75 liters are antifreeze and 25 liters are water.

→ More important: Antifreeze kills you! Don't drink it!

Ex Your boss makes $100,000 a year. You make 8% of his yearly salary. (You are also smarter, nicer, and better looking) And you work harder than him! How much do you make a year?

$\frac{8}{100} 100{,}000 = \$8{,}000$

→ you should get a new job!

Ex You take a job that pays $10/hour. You get a 10% raise each year. What is your hourly wage 2 years from now?

→ Not $12/hour

1st year: $\frac{10}{100} \cdot 10 = \$1$/hour raise ⇒ wage = $\$11$/hr

2nd year: $\frac{10}{100} \cdot 11 = \$1.10$/hour raise ⇒ wage = $\$12.10$/hour

Ex 12 is what percent of 16?

→ think: unknown percent of 16 is equal to 12

$$\frac{x}{100} \cdot 16 = 12 \Rightarrow x \cdot 16 = 12 \cdot 100$$

$$\Rightarrow x = \frac{12}{16} \cdot 100 = 75\%$$

Ex The original price of a shirt is $20. The sale price is $16. What is the percent change in the price?

→ % change = $\frac{\text{change}}{\text{original}} \cdot 100$

⇒ % change = $\frac{4}{20} \cdot 100 = 20\%$