1.2 - Operations with Real Numbers

→ Now that we've defined real numbers, we want to do operations with them.

→ 4 basic operations of arithmetic:
addition, subtraction, multiplication, & division

Addition

→ when we add numbers with the same sign:
↳ add the numbers and keep the sign.

Ex: a) $-12 + (-3) = -15$

b) $3.2 + 4.13 = 7.33$ → remember to line up the decimal point

→ when we add numbers with different signs:
↳ subtract the smaller absolute value from the larger absolute value and keep the sign of the larger absolute value.

Ex: a) $-12 + 6 = -6$

b) $2.1 + (-4.3) = -2.2$

Subtraction

→ Think of subtraction as adding a negative

Ex: a) $-4 - 2 = -4 + (-2) = -6$

b) $3.1 - 2.82 = 3.1 + (-2.82) = 0.28$

→ line up decimal

$$\begin{array}{r} 3.10 \\ -2.82 \\ \hline 0.28 \end{array}$$

→ what about adding and subtracting fractions?
↳ we need a common denominator

Fractions

$\frac{a}{b}$ → numerator, → denominator

→ when we have a common denominator:
  ↳ Add the numerators + keep the denominator
  ↳ simplify

→ when we don't have a common denominator
  ↳ Get a common denominator!
  ↳ Add the numerators & keep the denominator
  ↳ simplify

Ex a) $\frac{2}{3} - \frac{4}{3} = \frac{2}{3} + \frac{-4}{3} = \frac{2-4}{3} = \boxed{\frac{-2}{3}}$

b) $\frac{2}{3} - \frac{4}{5} = \frac{2}{3} + \frac{-4}{5} =$

↳ need a common denominator. How?

→ Main rule: $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$

↳ so then $\frac{2}{3} = \frac{2 \cdot 5}{3 \cdot 5} = \frac{10}{15}$ and $\frac{-4}{5} = \frac{(-4)(3)}{5(3)} = \frac{-12}{15}$

Thus $\frac{2}{3} - \frac{4}{5} = \frac{2}{3} + \frac{-4}{5} = \frac{10}{15} + \frac{-12}{15} = \frac{10-12}{15} = \boxed{\frac{-2}{15}}$

Ex c) $-\frac{1}{8} + \frac{2}{6} = \frac{(-1)(3)}{8 \cdot 3} + \frac{2 \cdot 4}{6 \cdot 4} = \frac{-3}{24} + \frac{8}{24} = \frac{-3+8}{24} = \boxed{\frac{5}{24}}$

→ What about mixed numbers like $2\frac{1}{4}$?
  ↳ make them into normal fractions!
  ↳ $2\frac{1}{4} = \frac{2 \cdot 4 + 1}{4} = \frac{8+1}{4} = \frac{9}{4}$

Ex a) $2\frac{1}{2} + 1\frac{2}{3} = \frac{2 \cdot 2 + 1}{2} + \frac{1 \cdot 3 + 2}{3} = \frac{4+1}{2} + \frac{3+2}{3} = \frac{5}{2} + \frac{5}{3}$

$= \frac{5 \cdot 3}{2 \cdot 3} + \frac{5 \cdot 2}{3 \cdot 2}$

$= \frac{15}{\quad} + \frac{10}{\quad} = \boxed{\frac{25}{\quad}}$

Multiplication

→ same sign gives positive output
→ opposite signs gives negative output

Ex: a) $2 \cdot 4 = 8$ → same sign

b) $(-3)(5) = -15$ → opposite signs

c) $(-2)(-3)(1)(-2) = (6)(1)(-2)$ → same signs

$= (6)(-2)$ → same signs

$= -12$ → opposite signs

d) $-4 \cdot 5 \cdot (-2) = -20 \cdot (-2)$ → opposite signs

$= 40$ → same signs

→ what about fractions?

↳ multiply numerators and denominators

Ex: a) $\left(\frac{2}{3}\right)\left(-\frac{3}{4}\right) = -\frac{2(3)}{3(4)} = -\frac{6}{12} = -\frac{1 \cdot 6}{2 \cdot 6} = -\frac{1}{2}$

b) $\left(\frac{5}{8}\right)\left(-\frac{3}{10}\right) = -\frac{5(3)}{8(10)} = -\frac{15}{80} = -\frac{3 \cdot 5}{16 \cdot 5} = -\frac{3}{16}$

Don't forget to simplify

Division → never divide by zero!

→ Multiply by the reciprocal. → swap numerator and denominator

↳ reciprocal of $4/5$? → $5/4$

↳ reciprocal of 2? $2 = \frac{2}{1}$ so reciprocal is $\frac{1}{2}$

Ex a) $10 \div 2 = \frac{10}{1} \cdot \frac{1}{2} = \frac{10}{2} = \frac{5 \cdot 2}{1 \cdot 2} = \frac{5}{1} = 5$

b) $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \cdot \frac{4}{3} = \frac{4}{6} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{2}{3}$

c) $\frac{1}{8} \div 2\frac{1}{2} = \frac{1}{8} \div \frac{2 \cdot 3 + 1}{2} = \frac{1}{8} \div \frac{6+1}{2} = \frac{1}{8} \div \frac{7}{2} = \frac{1}{8} \cdot \frac{2}{7} = \frac{2}{56}$

Ex d) $\dfrac{\left(\frac{2}{3}\right)}{\left(\frac{2}{5}\right)} = \frac{2}{3} \div \frac{2}{5} = \frac{2}{3} \cdot \frac{5}{2} = \frac{10}{6} = \frac{5 \cdot 2}{3 \cdot 2} = \frac{5}{3}$

↳ or $\dfrac{\frac{2}{3}}{\frac{2}{5}}$ → parentheses implied by big fraction bar

Note: We could have cancelled in a lot of the previous examples

↳ $\frac{\cancel{2}^{1}}{3} \cdot \frac{5}{\cancel{2}_{1}} = \frac{5}{3}$; ↳ $\frac{\cancel{2}^{1}}{\cancel{3}^{1}} \cdot \frac{-\cancel{3}^{-1}}{\cancel{4}_{2}} = -\frac{1}{2}$

Exponents

→ Multiply a number by itself multiple times.

Ex: a) $2^3 = 2 \cdot 2 \cdot 2 = 8$ → 2 is the base, → 3 is the exponent

b) $\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1 \cdot 1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{16}$

c) $-3^3 = -(3)(3)(3) = -27$ → negative not included

d) $(-2)^4 = (-2)(-2)(-2)(-2) = 16$ → negative included
↳ even # of negatives gives positive result

e) $(-2)^3 = (-2)(-2)(-2) = -8$ → odd # of negatives gives negative result

Order of Operations

P - Parentheses
E - Exponents
MD - Mult./division left to right
AS - Add/subtract left to right

Ex a) $24/8 - 4/2 = 3 - 2 = 1$

b) $(24/8 - 4)/2 = (3 - 4)/2 = -1/2$

c) $24/(8 - 4)/2 = 24/4/2 = 6/2 = 3$

d) $24/(8 - 4/2) = 24/(8 - 2) = 24/6 = 4$

Ex a) $2+3\cdot4^2 = 2+3\cdot16 = 2+48 = 50$

b) $(2+3)4^2 = 5\cdot4^2 = 5\cdot16 = 80$

c) $2+(3\cdot4)^2 = 2+12^2 = 2+144 = 146$

d) $(2+3\cdot4)^2 = (2+12)^2 = 14^2 = 196$

→ If we divide by zero, the world ends in flames and chaos! maybe...

Suppose $a=b$. Then we'll prove that $2=1$!

1.	$a^2 = ab$	→ multiply by a
2.	$a^2-b^2 = ab-b^2$	→ subtract b^2
3.	$(a-b)(a+b) = b(a-b)$	→ factor
4.	$a+b = b$	→ divide by $a-b$
5.	$b+b = b$	→ use $a=b$
6.	$2b = b$	→ simplify
7.	$2 = 1$	→ divide by b

What went wrong? When we divided by $a-b$ in step 4, we really divided by zero since $a=b$. Ahhhh! Flames and chaos!

→ If you value the world, don't ever divide by zero!