Math1100 Final Review Problems
Summer, 2007
Kelly MacArthur

You can bring a 8.5x11 inch paper of notes (on both sides, if you want) to use as a reference.

1. From attached pages:
We've exhausted most of the review problems from your book, so I took these problems from a different book (and included them in this assignment for you). Thus, these are NOT from your book.

Pg 723-727 #1-6 all, 7-19 odd, 23-30 all, 31-37 odd, 51-59 odd, 63-83 odd, 91-97 odd

pg 802-805 #1-23 odd, 31-35 odd, 47, 49

pg 858 #1-21 odd

pg 916 #1-25 odd

pg 1001 #5-21 odd

pg 1056 #1-11 odd, 15, 17

2. From your book:

Chapter 7 Review (pg 461) #11-15 odd

Chapter 8 Review (pg 530-531) #1, 2, 7, 8, 13

Section	Key Terms	Formula
9.4	Powers of x Rule	$\frac{d(x^n)}{dx} = nx^{n-1}$
	Constant Function Rule	$\frac{d(c)}{dx} = 0$ for constant c
	Coefficient Rule	$\frac{d}{dx}[c \cdot f(x)] = c \cdot f'(x)$
	Sum Rule	$\frac{d}{dx}[u + v] = \frac{du}{dx} + \frac{dv}{dx}$
	Difference Rule	$\frac{d}{dx}[u - v] = \frac{du}{dx} - \frac{dv}{dx}$
9.5	Product Rule	$\frac{d}{dx}[uv] = uv' + vu'$
	Quotient Rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$
9.6	Chain Rule	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
	Power Rule	$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$
9.8	Second derivative; third derivative; higher-order derivatives	
9.9	Marginal cost function	$\overline{MC} = C'(x)$
	Marginal revenue function	$\overline{MR} = R'(x)$
	Marginal profit function	$\overline{MP} = P'(x)$

REVIEW EXERCISES *Additional Practice with Guided Solutions on CD-ROM*

Section 9.1

In Problems 1–6, use the graph of $y = f(x)$ in Figure 9.42 to find the functional values and limits, if they exist.

1. (a) $f(-2)$ (b) $\lim_{x \to -2} f(x)$
2. (a) $f(-1)$ (b) $\lim_{x \to -1} f(x)$
3. (a) $f(4)$ (b) $\lim_{x \to 4^-} f(x)$
4. (a) $\lim_{x \to 4^+} f(x)$ (b) $\lim_{x \to 4} f(x)$
5. (a) $f(1)$ (b) $\lim_{x \to 1} f(x)$
6. (a) $f(2)$ (b) $\lim_{x \to 2} f(x)$

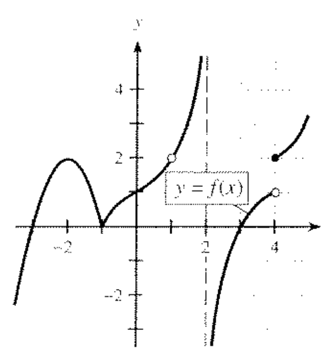


Figure 9.42

In Problems 7–20, find each limit, if it exists.

7. $\lim_{x\to 4}(3x^2 + x + 3)$

8. $\lim_{x\to 4}\dfrac{x^2 - 16}{x + 4}$

9. $\lim_{x\to -1}\dfrac{x^2 - 1}{x + 1}$

10. $\lim_{x\to 3}\dfrac{x^2 - 9}{x - 3}$

11. $\lim_{x\to 2}\dfrac{4x^3 - 8x^2}{4x^3 - 16x}$

12. $\lim_{x\to -\frac{1}{2}}\dfrac{x^2 - \frac{1}{4}}{6x^2 + x - 1}$

13. $\lim_{x\to 3}\dfrac{x^2 - 16}{x - 3}$

14. $\lim_{x\to -3}\dfrac{x^2 - 9}{x - 3}$

15. $\lim_{x\to 1}\dfrac{x^2 - 9}{x - 3}$

16. $\lim_{x\to 2}\dfrac{x^2 - 8}{x - 2}$

17. $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$

18. $\lim_{x\to -2} f(x)$ where $f(x) = \begin{cases} x^3 - x & \text{if } x < -2 \\ 2 - x^2 & \text{if } x \geq -2 \end{cases}$

19. $\lim_{h\to 0}\dfrac{3(x + h)^2 - 3x^2}{h}$

20. $\lim_{h\to 0}\dfrac{[(x + h) - 2(x + h)^2] - (x - 2x^2)}{h}$

In Problems 21 and 22, use tables to investigate each limit. Check your result analytically or graphically.

21. $\lim_{x\to 2}\dfrac{x^2 + 10x - 24}{x^2 - 5x + 6}$

22. $\lim_{x\to -\frac{1}{2}}\dfrac{x^2 + \frac{1}{6}x - \frac{1}{6}}{x^2 + \frac{5}{6}x + \frac{1}{6}}$

Section 9.2

Use the graph of $y = f(x)$ in Figure 9.42 on page 723 to answer the questions in Problems 23 and 24.

23. Is $f(x)$ continuous at
    (a) $x = -1$? (b) $x = 1$?

24. Is $f(x)$ continuous at
    (a) $x = -2$? (b) $x = 2$?

In Problems 25–30, suppose that

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 0 \\ x & \text{if } 0 < x < 1 \\ 2x^2 - 1 & \text{if } x \geq 1 \end{cases}$$

25. What is $\lim_{x\to -1} f(x)$?
26. What is $\lim_{x\to 0} f(x)$, if it exists?
27. What is $\lim_{x\to 1} f(x)$, if it exists?
28. Is $f(x)$ continuous at $x = 0$?
29. Is $f(x)$ continuous at $x = 1$?
30. Is $f(x)$ continuous at $x = -1$?

For the functions in Problems 31–34, determine which are continuous. Identify discontinuities for those that are not continuous.

31. $y = \dfrac{x^2 + 25}{x - 5}$

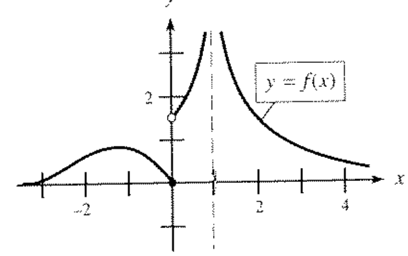
32. $y = \dfrac{x^2 - 3x + 2}{x - 2}$

33. $f(x) = \begin{cases} x + 2 & \text{if } x \leq 2 \\ 5x - 6 & \text{if } x > 2 \end{cases}$

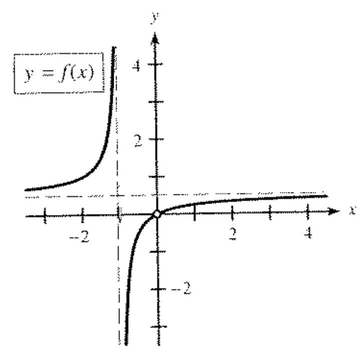
34. $y = \begin{cases} x^4 - 3 & \text{if } x \leq 1 \\ 2x - 3 & \text{if } x > 1 \end{cases}$

In Problems 35 and 36, use the graphs to find (a) the points of discontinuity, (b) $\lim_{x\to +\infty} f(x)$, and (c) $\lim_{x\to -\infty} f(x)$.

35.



36.



In Problems 37 and 38 evaluate the limits, if they exist.

37. $\lim_{x\to -\infty}\dfrac{2x^2}{1 - x^2}$

38. $\lim_{x\to +\infty}\dfrac{3x^{2/3}}{x + 1}$

Section 9.3

39. Find the average rate of change of

$$f(x) = 2x^4 - 3x + 7 \text{ over } [-1, 2].$$

In Problems 40 and 41, decide whether the statements are true or false.

40. $\lim_{h\to 0}\dfrac{f(x + h) - f(x)}{h}$ gives the formula for the slope of the tangent and the instantaneous rate of change of $f(x)$ at any value of x.

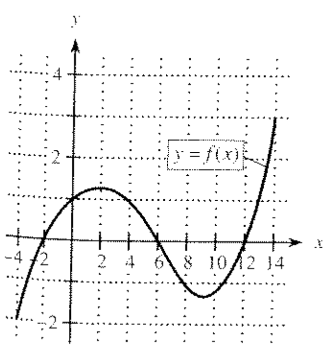
41. $\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ gives the equation of the tangent line to $f(x)$ at $x = c$.
42. Use the definition of derivative to find $f'(x)$ for $f(x) = 3x^2 + 2x - 1$.
43. Use the definition of derivative to find $f'(x)$ if $f(x) = x - x^2$.

Use the graph of $y = f(x)$ in Figure 9.42 on page 723 to answer the questions in Problems 44–46.

44. Explain which is greater: the average rate of change of f over $[-3, 0]$ or over $[-1, 0]$.
45. Is $f(x)$ differentiable at
    (a) $x = -1$? (b) $x = 1$?
46. Is $f(x)$ differentiable at
    (a) $x = -2$? (b) $x = 2$?
47. Let $f(x) = \frac{\sqrt[3]{4x}}{(3x^2 - 10)^2}$. Approximate $f'(2)$
    (a) by using the numerical derivative feature of a graphing utility, and
    (b) by evaluating $\frac{f(2+h) - f(2)}{h}$ with $h = 0.0001$.
48. Use the given table of values for $g(x)$ to
    (a) find the average rate of change of $g(x)$ over $[2,5]$.
    (b) approximate $g'(4)$ as accurately as possible.

x	2	2.3	3.1	4	4.3	5
$g(x)$	13.2	12.1	9.7	12.2	14.3	18.1

Use the following graph of $f(x)$ to complete Problems 49 and 50.



49. Estimate $f'(4)$.
50. Rank the following from smallest to largest and explain.
    A: $f'(2)$ B: $f'(6)$
    C: the average rate of change over $[2, 10]$

Section 9.4

51. If $c = 4x^5 - 6x^3$, find c'.
52. If $f(x) = 4x^2 - 1$, find $f'(x)$.
53. If $p = 3q + \sqrt{7}$, find dp/dq.
54. If $y = \sqrt{x}$, find y'.
55. If $f(z) = \sqrt[3]{z^4}$, find $f'(z)$.
56. If $v(x) = 4/\sqrt[3]{x}$, find $v'(x)$.
57. If $y = \frac{1}{x} - \frac{1}{\sqrt{x}}$, find y'.
58. If $f(x) = \frac{3}{2x^2} - \sqrt[3]{x} + 4^5$, find $f'(x)$.
59. Write the equation of the line tangent to the graph of $y = 3x^5 - 6$ at $x = 1$.
60. Write the equation of the line tangent to the curve $y = 3x^3 - 2x$ at the point where $x = 2$.

In Problems 61 and 62, (a) find all x-values where the slope of the tangent equals zero, (b) find points (x, y) where the slope of the tangent equals zero, and (c) use a graphing utility to graph the function and label the points found in (b).

61. $f(x) = x^3 - 3x^2 + 1$
62. $f(x) = x^6 - 6x^4 + 8$

Section 9.5

63. If $f(x) = (3x - 1)(x^2 - 4x)$, find $f'(x)$.
64. Find y' if $y = (x^2 + 1)(3x^3 + 1)$.
65. If $p = \frac{2q - 1}{q^2}$, find $\frac{dp}{dq}$.
66. Find $\frac{ds}{dt}$ if $s = \frac{\sqrt{t}}{(3t + 1)}$.
67. Find $\frac{dy}{dx}$ for $y = \sqrt{x}(3x + 2)$.
68. Find $\frac{dC}{dx}$ for $C = \frac{5x^4 - 2x^2 + 1}{x^3 + 1}$.

Section 9.6

69. If $y = (x^3 - 4x^2)^3$, find y'.
70. If $y = (5x^6 + 6x^4 + 5)^6$, find y'.
71. If $y = (2x^4 - 9)^9$, find $\frac{dy}{dx}$.
72. Find $g'(x)$ if $g(x) = \frac{1}{\sqrt{x^3 - 4x}}$.

Section 9.7

73. Find $f'(x)$ if $f(x) = x^2(2x^4 + 5)^8$.

74. Find S' if $S = \dfrac{(3x + 1)^2}{x^2 - 4}$.

75. Find $\dfrac{dy}{dx}$ if $y = [(3x + 1)(2x^3 - 1)]^{12}$.

76. Find y' if $y = \left(\dfrac{x + 1}{1 - x^2}\right)^3$.

77. Find y' if $y = x\sqrt{x^2 - 4}$.

78. Find $\dfrac{dy}{dx}$ if $y = \dfrac{x}{\sqrt[3]{3x - 1}}$.

Section 9.8

In Problems 79 and 80, find the second derivatives.

79. $y = \sqrt{x} - x^2$ 80. $y = x^4 - \dfrac{1}{x}$

In Problems 81 and 82, find the fifth derivatives.

81. $y = (2x + 1)^4$

82. $y = \dfrac{(1 - x)^6}{24}$

83. If $\dfrac{dy}{dx} = \sqrt{x^2 - 4}$, find $\dfrac{d^3y}{dx^3}$.

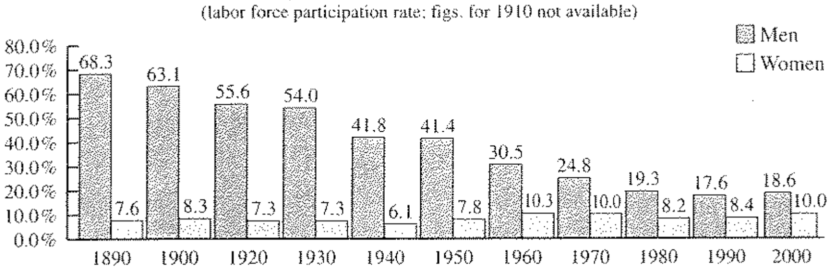
84. If $\dfrac{d^2y}{dx^2} = \dfrac{x}{x^2 + 1}$, find $\dfrac{d^4y}{dx^4}$.

APPLICATIONS

Section 9.3

Elderly in the work force **Problems 85 and 86 use the following graph, which shows the percentage of elderly men and women in the work force for selected years from 1890 to 2000.**

Elderly in the Labor Force, 1890-2000
(labor force participation rate; figs. for 1910 not available)



Source: Bureau of the Census, U.S. Department of Commerce

85. For the period from 1900 to 2000, find and interpret the annual average rate of change of
 (a) elderly men in the work force and
 (b) elderly women in the work force.

86. (a) During what decade was there an increase in the percentage of elderly men in the work force? Find and interpret the annual average rate of change for elderly men in that decade.
 (b) During what decade was there the largest increase in the percentage of elderly women in the work force? Find and interpret the average rate of change for elderly women in that decade.

Section 9.4

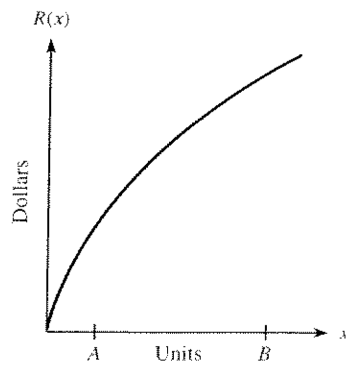
87. ***Demand*** Suppose that the demand for x units of a product is given by $x = (100/p) - 1$, where p is the price per unit of the product. Find and interpret the rate of change of demand with respect to price if the price is
 (a) \$10. (b) \$20.

88. ***Voter turnout*** By using data from the Federal Election Commission, the percent of eligible voters who voted in Presidential elections can be modeled by

$$P(x) = 64.37x^{-0.0917}$$

where x is the number of presidential elections past 1952. Find and interpret
 (a) $P(12)$ (b) $P'(12)$
 (c) This model predicts that P will decrease. What happened in the 2000 election (when $x = 12$) that may change that?

89. ***Revenue*** The following graph shows the revenue function for a commodity. Will the $(A + 1)$st item sold or the $(B + 1)$st item sold produce more revenue? Explain.



Section 9.6

90. ***Demand*** The demand q for a product at price p is given by

$$q = 10{,}000 - 50\sqrt{0.02p^2 + 500}$$

Find the rate of change of demand with respect to price.

91. ***Supply*** The number of units x of a product that is supplied at price p is given by

$$x = \sqrt{p - 1}, \quad p \geq 1$$

If the price p is \$10, what is the rate of change of the supply with respect to the price and what does it tell us?

Section 9.9

In Problems 92–99, cost, revenue, and profit are in dollars and x is the number of units.

92. ***Cost*** If the cost function for a particular good is $C(x) = 3x^2 + 6x + 600$, what is the
    (a) marginal cost function?
    (b) marginal cost if 30 units are produced?
    (c) interpretation of your answer in part (b)?

93. ***Cost*** If the total cost function for a commodity is $C(x) = 400 + 5x + x^3$, what is the marginal cost when 4 units are produced and what does it mean?

94. ***Revenue*** The total revenue function for a commodity is $R = 40x - 0.02x^2$, with x representing the number of units.
    (a) Find the marginal revenue function.
    (b) At what level of production will marginal revenue be 0?

95. ***Profit*** If the total revenue function for a product is given by $R(x) = 60x$ and the total cost function is given by $C = 200 + 10x + 0.1x^2$, what is the marginal profit at $x = 10$? What does the marginal profit at $x = 10$ predict?

96. ***Revenue*** The total revenue function for a commodity is given by $R = 80x - 0.04x^2$.
    (a) Find the marginal revenue function.
    (b) What is the marginal revenue at $x = 100$?
    (c) Interpret your answer in part (b).

97. ***Revenue*** If the revenue function for a product is

$$R(x) = \frac{60x^2}{2x + 1}$$

find the marginal revenue.

98. ***Profit*** A firm has monthly costs given by

$$C = 45{,}000 + 100x + x^3$$

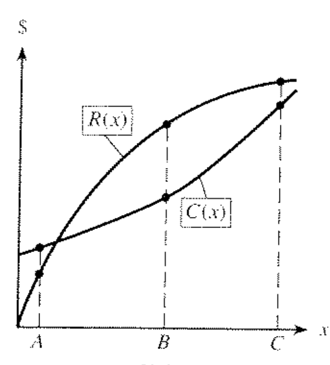
where x is the number of units produced per month. The firm can sell its product in a competitive market for \$4600 per unit. Find the marginal profit.

99. ***Profit*** A small business has weekly costs of

$$C = 100 + 30x + \frac{x^2}{10}$$

where x is the number of units produced each week. The competitive market price for this business's product is \$46 per unit. Find the marginal profit.

100. ***Cost, revenue, and profit*** The following graph shows the total revenue and total cost functions for a company. Use the graph to decide (and justify) at which of points A, B, and C
    (a) the cost of the next item will be least.
    (b) the profit will be greatest.
    (c) the profit from the sale of the next item will be greatest.
    (d) the next item sold will reduce the profit.



Section	Key Terms	Formula
11.3	Implicit differentiation	
11.4	Related rates Percentage rates of change	
11.5	Elasticity of demand	$\eta = \frac{-p}{q} \cdot \frac{dq}{dp}$
	Elastic	$\eta > 1$
	Inelastic	$\eta < 1$
	Unitary elastic	$\eta = 1$
	Taxation in competitive market	
	Supply function after taxation	$p = f(q) + t$

REVIEW EXERCISES *Additional Practice with Guided Solutions on CD-ROM*

Sections 11.1 and 11.2

In Problems 1–12, find the indicated derivative.

1. If $y = e^{3x^2 - x}$, find dy/dx.
2. If $y = \ln e^{x^2}$, find y'.
3. If $p = \ln\left(\frac{q}{q^2 - 1}\right)$, find $\frac{dp}{dq}$.
4. If $y = xe^{x^2}$, find dy/dx.
5. If $f(x) = 5e^{2x} - 40e^{-0.1x} + 11$, find $f'(x)$.
6. If $g(x) = (2e^{3x+1} - 5)^3$, find $g'(x)$.
7. If $y = \ln(3x^4 + 7x^2 - 12)$, find dy/dx.
8. If $s = \frac{3}{4}\ln(x^{12} - 2x^4 + 5)$, find ds/dx.
9. If $y = 3^{3x-4}$, find dy/dx.
10. If $y = 1 + \log_8(x^{10})$, find dy/dx.
11. If $y = \frac{\ln x}{x}$, find $\frac{dy}{dx}$.
12. If $y = \frac{1 + e^{-x}}{1 - e^{-x}}$, find $\frac{dy}{dx}$.
13. Write the equation of the line tangent to $y = 4e^{x^3}$ at $x = 1$.
14. Write the equation of the line tangent to $y = x \ln x$ at $x = 1$.

Section 11.3

In Problems 15–20, find the indicated derivative.

15. If $y \ln x = 5y$, find dy/dx.
16. Find dy/dx for $e^{xy} = y$.
17. Find dy/dx for $y^2 = 4x - 1$.
18. Find dy/dx if $x^2 + 3y^2 + 2x - 3y + 2 = 0$.
19. Find dy/dx for $3x^2 + 2x^3y^2 - y^5 = 7$.
20. Find the second derivative y'' if $x^2 + y^2 = 1$.
21. Find the slope of the tangent to the curve $x^2 + 4x - 3y^2 + 6 = 0$ at $(3, 3)$.
22. Find the points where tangents to the graph of the equation in Problem 21 are horizontal.

Section 11.4

23. Suppose $3x^2 - 2y^3 = 10y$, where x and y are differentiable functions of t. If $dx/dt = 2$, find dy/dt when $x = 10$ and $y = 5$.
24. A right triangle with legs of lengths x and y has its area given by

$$A = \frac{1}{2}xy$$

If the rate of change of x is 2 units per minute and the rate of change of y is 5 units per minute, find the rate of change of the area when $x = 4$ and $y = 1$.

APPLICATIONS

Section 11.1

25. ***Deforestation*** One of the major causes for rain forest deforestation is agricultural and residential development. The number of hectares destroyed in a particular year t can be modeled by

$$y = -3.91435 + 2.62196 \ln t$$

where $t = 0$ in 1950.

Section	Key Terms	Formula
12.4	Total cost	$C(x) = \int \overline{MC}\, dx$
	Total revenue	$R(x) = \int \overline{MR}\, dx$
	Profit	$P(x) = R(x) - C(x)$
	Marginal propensity to consume	$\dfrac{dC}{dy}$
	Marginal propensity to save	$\dfrac{dS}{dy} = 1 - \dfrac{dC}{dy}$
	National consumption	$C = \int f'(y)\, dy = \int \dfrac{dC}{dy}\, dy$
12.5	Differential equations	
	Solutions	
	General	
	Particular	
	First order	$\dfrac{dy}{dx} = f(x) \Rightarrow y = \int f(x)\, dx$
	Separable	$g(y)\, dy = f(x)\, dx \Rightarrow \int g(y)\, dy = \int f(x)\, dx$
	Radioactive decay	$\dfrac{dy}{dt} = ky$
	Drugs in an organ	Rate = (rate in) − (rate out)

REVIEW EXERCISES *Additional Practice with Guided Solutions on CD-ROM*

Sections 12.1–12.3

Evaluate the integrals in Problems 1–26.

1. $\int x^6 dx$
2. $\int x^{1/2} dx$
3. $\int (x^3 - 3x^2 + 4x + 5)\, dx$
4. $\int (x^2 - 1)^2\, dx$
5. $\int (x^2 - 1)^2 x\, dx$
6. $\int (x^3 - 3x^2)^5 (x^2 - 2x)\, dx$
7. $\int (x^3 + 4)^2 x\, dx$
8. $\int (x^3 + 4)^6 x^2\, dx$
9. $\int \dfrac{x^2}{x^3 + 1}\, dx$
10. $\int \dfrac{x^2}{(x^3 + 1)^2}\, dx$
11. $\int \dfrac{x^2\, dx}{\sqrt[3]{x^3 - 4}}$
12. $\int \dfrac{x^2\, dx}{x^3 - 4}$
13. $\int \dfrac{x^3 + 1}{x^2}\, dx$
14. $\int \dfrac{x^3 - 3x + 1}{x - 1}\, dx$
15. $\int y^2 e^{y^3} dy$
16. $\int (x - 1)^2\, dx$
17. $\int \dfrac{3x^2}{2x^3 - 7}\, dx$
18. $\int \dfrac{5\, dx}{e^{4x}}$
19. $\int (x^3 - e^{3x})\, dx$
20. $\int x e^{1 + x^2}\, dx$
21. $\int \dfrac{6x^7}{(5x^8 + 7)^3}\, dx$
22. $\int \dfrac{7x^3}{\sqrt{1 - x^4}}\, dx$
23. $\int \left(\dfrac{e^{2x}}{2} + \dfrac{2}{e^{2x}}\right) dx$
24. $\int \left[x - \dfrac{1}{(x + 1)^2}\right] dx$
25. (a) $\int (x^2 - 1)^4 x\, dx$ (b) $\int (x^2 - 1)^{10} x\, dx$
    (c) $\int (x^2 - 1)^7 3x\, dx$ (d) $\int (x^2 - 1)^{-2/3} x\, dx$
26. (a) $\int \dfrac{2x\, dx}{x^2 - 1}$ (b) $\int \dfrac{2x\, dx}{(x^2 - 1)^2}$
    (c) $\int \dfrac{3x\, dx}{\sqrt{x^2 - 1}}$ (d) $\int \dfrac{3x\, dx}{x^2 - 1}$

Section 12.5

In Problems 27–32, find the general solution to each differential equation.

27. $\dfrac{dy}{dt} = 4.6e^{-0.05t}$
28. $dy = (64 + 76x - 36x^2)\, dx$
29. $\dfrac{dy}{dx} = \dfrac{4x}{y - 3}$
30. $t\, dy = \dfrac{dt}{y + 1}$
31. $\dfrac{dy}{dx} = \dfrac{x}{e^y}$
32. $\dfrac{dy}{dt} = \dfrac{4y}{t}$

Section	Key Terms	Formula
	Simpson's Rule for $\int_a^b f(x)\,dx$	$\approx \frac{h}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$, where n is even and $h = \frac{b-a}{n}$
	Error formula	$\lvert E \rvert \le \frac{(b-a)^5}{180n^4}\left[\max_{a\le x\le b} \lvert f^{(4)}(x)\rvert\right]$

REVIEW EXERCISES *Additional Practice with Guided Solutions on CD-ROM*

Section 13.1

1. Calculate $\sum_{k=1}^{8} (k^2 + 1)$.
2. Use formulas to simplify

$$\sum_{i=1}^{n} \frac{3i}{n^3}$$

3. Use 6 subintervals of the same size to approximate the area under the graph of $y = 3x^2$ from $x = 0$ to $x = 1$. Use the right-hand endpoints of the subintervals to find the heights of the rectangles.
4. Use rectangles to find the area under the graph of $y = 3x^2$ from $x = 0$ to $x = 1$. Use n equal subintervals.

Section 13.2

5. Use a definite integral to find the area under the graph of $y = 3x^2$ from $x = 0$ to $x = 1$.
6. Find the area between the graph of $y = x^3 - 4x + 5$ and the x-axis from $x = 1$ to $x = 3$.

Evaluate the integrals in Problems 7–18.

7. $\int_1^4 4\sqrt{x^3}\,dx$
8. $\int_{-3}^{2} (x^3 - 3x^2 + 4x + 2)\,dx$
9. $\int_0^5 (x^3 + 4x)\,dx$
10. $\int_{-2}^{3} (x + 2)^2\,dx$
11. $\int_{-3}^{-1} (x + 1)\,dx$
12. $\int_2^3 \frac{x^2}{2x^3 - 7}\,dx$
13. $\int_{-1}^{2} (x^2 + x)\,dx$
14. $\int_1^4 \left(\frac{1}{x} + \sqrt{x}\right) dx$
15. $\int_0^4 (2x + 1)^{1/2}\,dx$
16. $\int_0^1 \frac{x}{x^2 + 1}\,dx$
17. $\int_0^1 e^{-2x}\,dx$
18. $\int_0^1 xe^{x^2}\,dx$

Section 13.3

Find the area between the curves in Problems 19–22.

19. $y = x^2 - 3x + 2$ and $y = x^2 + 4$ from $x = 0$ to $x = 5$
20. $y = x^2$ and $y = 4x + 5$
21. $y = x^3$ and $y = x$ from $x = -1$ to $x = 0$
22. $y = x^3 - 1$ and $y = x - 1$

Section 13.5

Evaluate the integrals in Problems 23–26, using Table 13.2.

23. $\int \sqrt{x^2 - 4}\,dx$
24. $\int_0^1 3^x\,dx$
25. $\int x \ln x^2\,dx$
26. $\int \frac{dx}{x(3x + 2)}$

Section 13.6

In Problems 27–30, use integration by parts to evaluate.

27. $\int x^5 \ln x\,dx$
28. $\int xe^{-2x}\,dx$
29. $\int \frac{x\,dx}{\sqrt{x + 5}}$
30. $\int_1^e \ln x\,dx$

REVIEW EXERCISES *Additional Practice with Guided Solutions on CD-ROM*

Section 14.1

1. What is the domain of $z = \dfrac{3}{2x - y}$?
2. What is the domain of $z = \dfrac{3x + 2\sqrt{y}}{x^2 + y^2}$?
3. If $w(x, y, z) = x^2 - 3yz$, find $w(2, 3, 1)$.
4. If $Q(K, L) = 70K^{2/3}L^{1/3}$, find $Q(64{,}000, 512)$.

Section 14.2

5. Find $\dfrac{\partial z}{\partial x}$ if $z = 5x^3 + 6xy + y^2$.
6. Find $\dfrac{\partial z}{\partial y}$ if $z = 12x^5 - 14x^3y^3 + 6y^4 - 1$.

In Problems 7–12, find z_x and z_y.

7. $z = 4x^2y^3 + \dfrac{x}{y}$
8. $z = \sqrt{x^2 + 2y^2}$
9. $z = (xy + 1)^{-2}$
10. $z = e^{x^2y^3}$
11. $z = e^{xy} + y \ln x$
12. $z = e^{\ln xy}$
13. Find the partial derivative of $f(x, y) = 4x^3 - 5xy^2 + y^3$ with respect to x at the point $(1, 2, -8)$.
14. Find the slope of the tangent in the x-direction to the surface $z = 5x^4 - 3xy^2 + y^2$ at $(1, 2, -3)$.

In Problems 15–18, find the second partials.
(a) z_{xx} (b) z_{yy} (c) z_{xy} (d) z_{yx}

15. $z = x^2y - 3xy$
16. $z = 3x^3y^4 - \dfrac{x^2}{y^2}$
17. $z = x^2e^{y^2}$
18. $z = \ln(xy + 1)$

Section 14.4

19. Test $z = 16 - x^2 - xy - y^2 + 24y$ for maxima and minima.
20. Test $z = x^3 + y^3 - 3xy$ for maxima and minima.

Section 14.5

21. Find the minimum value of $z = 4x^2 + y^2$ subject to the constraint $x + y = 10$.
22. Find the maximum value of $z = x^4y^2$ subject to the constraint $x + y = 9$, $x \geq 0$, $y \geq 0$.

APPLICATIONS

Section 14.1

23. ***Utility*** Suppose that the utility function for two goods X and Y is given by $U = x^2y$, and a consumer purchases 6 units of X and 15 units of Y. If the consumer purchases 60 units of Y, how many units of X must be purchased to retain the same level of utility?
24. ***Utility*** Suppose that an indifference curve for two products, X and Y, has the equation $xy = 1600$. If 80 units of X are purchased, how many units of Y must be purchased?

Section 14.2

25. ***Concorde sonic booms*** The width of the region on the ground on either side of the path of France's Concorde jet in which people hear the sonic boom is given by

$$w = f(T, h, d) = 2\sqrt{Th/d}$$

where T is the air temperature at ground level in kelvins (K), h is the Concorde's altitude in kilometers, and d is the vertical temperature gradient (the temperature drop in kelvins per kilometer).*
 (a) Suppose the Concorde approaches Washington, D.C., from Europe on a course that takes it south of Nantucket Island at an altitude of 16.8 km. If the surface temperature is 293 K and the vertical temperature gradient is 5 K/km, how far south of Nantucket must the plane pass to keep the sonic boom off the island?
 (b) Interpret $f(287, 17.1, 4.9) \approx 63.3$.
 (c) Find $\dfrac{\partial f}{\partial h}(293, 16.8, 5)$ and interpret the result.
 (d) Find $\dfrac{\partial f}{\partial d}(293, 16.8, 5)$ and interpret the result.

Section 14.3

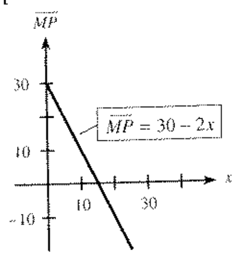
26. ***Cost*** The joint cost, in dollars, for two products is given by $C(x, y) = x^2\sqrt{y^2 + 13}$. Find the marginal cost with respect to
 (a) x if 20 units of x and 6 units of y are produced.
 (b) y if 20 units of x and 6 units of y are produced.

* N. K. Balachandra, W. L. Donn, and D. H. Rind, "Concorde Sonic Booms as an Atmospheric Probe," *Science*, 1 July 1977, Vol. 197, p. 47

29. **(a)** $P(x) = R(x) - C(x)$, so profit is the distance between $R(x)$ and $C(x)$ (when $R(x)$ is above $C(x)$). $P(100) < P(700) < P(400)$; $P(100) < 0$, so there is a loss when 100 units are sold.
(b) This asks us to rank $\overline{MP}(100)$, $\overline{MP}(400)$, and $\overline{MP}(700)$. Because $\overline{MP} = \overline{MR} - \overline{MC}$, compare the slopes of the tangents to $R(x)$ and $C(x)$ at the three x-values. Thus $\overline{MP}(700) < \overline{MP}(400) < \overline{MP}(100)$. $\overline{MP}(700) < 0$ because $C(x)$ is steeper than $R(x)$ at $x = 700$. At $x = 100$, $R(x)$ is much steeper than $C(x)$.

31. **(a)** $A < B < C$. Amount of profit is the height of the graph. There is a loss at A.
(b) $C < B < A$. Marginal profit is the slope of the tangent to the graph. Marginals (slopes) are positive at all three points.

33. **(a)**



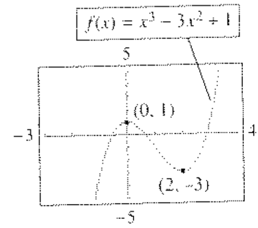
(b) 15 **(c)** 15 **(d)** \$25

35. 70 **37.** \$200

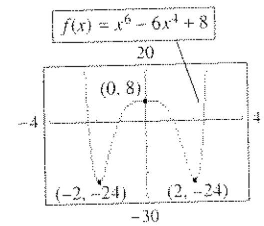
Chapter 9 Review Exercises *(page 723)*

1. **(a)** 2 **(b)** 2 **2.** **(a)** 0 **(b)** 0
3. **(a)** 2 **(b)** 1 **4.** **(a)** 2 **(b)** does not exist
5. **(a)** does not exist **(b)** 2
6. **(a)** does not exist **(b)** does not exist
7. 55 **8.** 0 **9.** -2 **10.** 6 **11.** $\frac{1}{2}$ **12.** $\frac{1}{5}$
13. no limit **14.** 0 **15.** 4 **16.** no limit
17. 3 **18.** no limit **19.** $6x$ **20.** $1 - 4x$
21. -14 **22.** 5 **23.** **(a)** yes **(b)** no
24. **(a)** yes **(b)** no **25.** 2 **26.** no limit
27. 1 **28.** no **29.** yes **30.** yes
31. discontinuity at $x = 5$ **32.** discontinuity at $x = 2$
33. continuous **34.** discontinuity at $x = 1$
35. **(a)** $x = 0$, $x = 1$ **(b)** 0 **(c)** 0
36. **(a)** $x = -1$, $x = 0$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{2}$
37. -2 **38.** 0 **39.** 7 **40.** true **41.** false
42. $f'(x) = 6x + 2$ **43.** $f'(x) = 1 - 2x$
44. $[-1, 0]$; the segment over this interval is steeper
45. **(a)** no **(b)** no **46.** **(a)** yes **(b)** no
47. **(a)** -5.9171 (to four decimal places) **(b)** -5.9
48. **(a)** 4.9/3 **(b)** 7 **49.** about $-1/4$
50. B, C, A: $B < 0$ and $C < 0$; the tangent line at $x = 6$ falls more steeply than the segment over $[2, 10]$.

51. $20x^4 - 18x^2$ **52.** $8x$ **53.** 3
54. $1/(2\sqrt{x})$ **55.** 0 **56.** $-4/(3\sqrt[3]{x^4})$
57. $\dfrac{-1}{x^2} + \dfrac{1}{2\sqrt{x^3}}$ **58.** $\dfrac{-3}{x^3} - \dfrac{1}{3\sqrt[3]{x^2}}$
59. $y = 15x - 18$ **60.** $y = 34x - 48$
61. **(a)** $x = 0$, $x = 2$ **(b)** $(0, 1)$ $(2, -3)$



62. **(a)** $x = 0$, $x = 2$, $x = -2$
(b) $(0, 8)$ $(2, -24)(-2, -24)$



63. $9x^2 - 26x + 4$ **64.** $15x^4 + 9x^2 + 2x$
65. $\dfrac{2(1 - q)}{q^3}$ **66.** $\dfrac{1 - 3t}{[2\sqrt{t}(3t + 1)^2]}$ **67.** $\dfrac{9x + 2}{2\sqrt{x}}$
68. $\dfrac{5x^6 + 2x^4 + 20x^3 - 3x^2 - 4x}{(x^3 + 1)^2}$
69. $(9x^2 - 24x)(x^3 - 4x^2)^2$
70. $6(30x^5 + 24x^3)(5x^6 + 6x^4 + 5)^5$
71. $72x^3(2x^4 - 9)^8$ **72.** $\dfrac{-(3x^2 - 4)}{2\sqrt{(x^3 - 4x)^3}}$
73. $2x(2x^4 + 5)^7(34x^4 + 5)$ **74.** $\dfrac{-2(3x + 1)(x + 12)}{(x^2 - 4)^2}$
75. $36[(3x + 1)(2x^3 - 1)]^{11}(8x^3 + 2x^2 - 1)$
76. $\dfrac{3}{(1 - x)^4}$ **77.** $\dfrac{(2x^2 - 4)}{\sqrt{x^2 - 4}}$ **78.** $\dfrac{2x - 1}{(3x - 1)^{4/3}}$
79. $y'' = \frac{-1}{4}x^{-3/2} - 2$ **80.** $y'' = 12x^2 - 2/x^3$
81. $\dfrac{d^5y}{dx^5} = 0$ **82.** $\dfrac{d^5y}{dx^5} = -30(1 - x)$
83. $\dfrac{d^3y}{dx^3} = -4/[(x^2 - 4)^{3/2}]$ **84.** $\dfrac{d^4y}{dx^4} = \dfrac{2x(x^2 - 3)}{(x^2 + 1)^3}$
85. The average annual percent change of **(a)** elderly men in the work force is -0.445% per year and of **(b)** elderly women in the work force is 0.017% per year.
86. **(a)** 1990–2000: Average rate = 0.10% per year for elderly men in the work force.
(b) 1950–1960: Average rate = 0.25% per year for elderly women in the work force.

87. **(a)** $x'(10) = -1$ means if price changes from \$10 to \$11, the number of units demanded will change by about -1.
(b) $x'(20) = -\frac{1}{4}$ means if price changes from \$20 to \$21, the number of units demanded will change by about $-\frac{1}{4}$.

88. **(a)** $P(12) = 51.253$ means that in 2000, the voter turnout was 51.253%.
(b) $P'(12) = -0.3917$ means in 2000, the rate of change of voter turnout is predicted to be -0.3917% per presidential election.
(c) Florida's vote was a virtual tie between Bush and Gore, and the importance of those electoral votes made it clear how important each vote is.

89. The slope of the tangent at A gives $\overline{MR}(A)$. The tangent line at A is steeper (so has greater slope) than the tangent line at B. Hence, $\overline{MR}(A) > \overline{MR}(B)$, so the $(A + 1)$ st unit brings more revenue.

90. $\dfrac{dq}{dp} = \dfrac{-p}{\sqrt{0.02p^2 + 500}}$

91. $x'(10) = \frac{1}{6}$ means if price changes from \$10 to \$11, the number of units supplied will change by about $\frac{1}{6}$.

92. **(a)** $\overline{MC} = 6x + 6$ **(b)** 186
(c) If a 31st unit is produced, costs will change by about \$186.

93. $C'(4) = 53$ means that a 5th unit produced would change total costs by about \$53.

94. **(a)** $\overline{MR} = 40 - 0.04x$ **(b)** $x = 1000$ units

95. $\overline{MP}(10) = 48$ means if an 11th unit is sold, profit will change by about \$48.

96. **(a)** $\overline{MR} = 80 - 0.08x$ **(b)** 72
(c) If a 101st unit is sold, revenue will change by about \$72.

97. $\dfrac{120x(x + 1)}{(2x + 1)^2}$ **98.** $\overline{MP} = 4500 - 3x^2$

99. $\overline{MP} = 16 - 0.2x$

100. **(a)** A: Tangent line to $C(x)$ has smallest slope at A, so $\overline{MC}(A)$ is smallest and the next item at A will cost least.
(b) B: $R(x) > C(x)$ at both B and C. Distance between $R(x)$ and $C(x)$ gives the amount of profit and is greatest at B.
(c) A: $\overline{MR}$ greatest at A and $\overline{MC}$ least at A, as seen from the slopes of the tangents. Hence $\overline{MP}(A)$ is greatest, so the next item at A will give the greatest profit.
(d) C: $\overline{MC}(C) > \overline{MR}(C)$, as seen from the slopes of the tangents. Hence $\overline{MP}(C) < 0$, so the next unit sold reduces profit.

Chapter 9 Test *(page 728)*

1. **(a)** $\frac{3}{4}$ **(b)** $-8/5$ **(c)** 9/8 **(d)** does not exist

2. **(a)** $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x + h) - f(x)}{h}$
(b) $f'(x)$

$$= \lim_{h\to 0} \frac{[3(x + h)^2 - (x + h) + 9] - [3x^2 - x + 9]}{h}$$
$$= \lim_{h\to 0} \frac{[3x^2 + 6xh + 3h^2 - x - h + 9] - [3x^2 - x + 9]}{h}$$
$$= \lim_{h\to 0} \frac{6xh + 3h^2 - h}{h} = \lim_{h\to 0}[6x + 3h - 1] = 6x - 1$$

3. $x = 0, x = 8$

4. **(a)** $\dfrac{99x^2 - 24x^9}{(2x^7 + 11)^2}$
(b) $(3x^5 - 2x + 3)(40x^9 + 40x^3) + (4x^{10} + 10x^4 - 17)(15x^4 - 2)$
(c) $9(10x^4 + 21x^2)(2x^5 + 7x^3 - 5)^{11}$
(d) $2(8x^2 + 5x + 18)(2x + 5)^5$
(e) $\dfrac{6}{\sqrt{x}} + \dfrac{20}{x^3}$

5. $\dfrac{d^3y}{dx^3} = 6 + 60x^{-6}$

6. **(a)** $y = -15x - 5$ **(b)** $(4, -90), (-2, 18)$

7. -15 **8.** **(a)** 2 **(b)** does not exist **(c)** -4

9. $g(-2) = 8$; $\lim\limits_{x\to -2^-} g(x) = 8$, $\lim\limits_{x\to -2^+} g(x) = -8$
$\therefore \lim\limits_{x\to -2} g(x)$ does not exist and $g(x)$ is not continuous at $x = -2$.

10. **(a)** $P(x) = 50x - 0.01x^2 - 10{,}000$
(b) $\overline{MP} = 50 - 0.02x$
(c) $\overline{MP}(1000) = 30$ means the predicted profit from the sale of the 1001st unit is approximately \$30.

11. 104

12. **(a)** -5 **(b)** -1 **(c)** 4 **(d)** does not exist
(e) 2 **(f)** $-4, 1, 3, 6$ **(g)** $-4, 3, 6$ **(h)** 3/2
(i) $f'(-2) <$ average rate over $[-2, 2] < f'(2)$

13. **(a)** $\frac{2}{3}$ **(b)** -4 **(c)** $\frac{2}{3}$

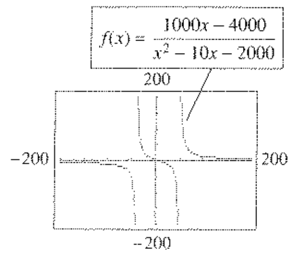
14. **(a)** B: $R(x) > C(x)$ at B, so there is profit. Distance between $R(x)$ and $C(x)$ gives the amount of profit.
(b) A: $C(x) > R(x)$
(c) A and B: slope of $R(x)$ is greater than the slope of $C(x)$. Hence $\overline{MR} > \overline{MC}$ and $\overline{MP} > 0$.
(d) C: Slope of $C(x)$ is greater than the slope of $R(x)$. Hence $\overline{MC} > \overline{MR}$ and $\overline{MP} < 0$.

Exercise 10.1 *(page 747)*

1. **(a)** (1, 5) **(b)** (4, 1) **(c)** (−1, 2)

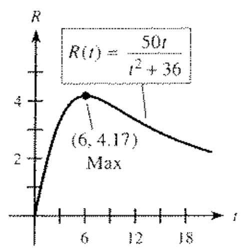
3. **(a)** (1, 5) **(b)** (4, 1) **(c)** (−1, 2)

(c) x: −200 to 200
y: −200 to 200



35. **(a)** none **(b)** $C \geq 0$ **(c)** $p = 100$ **(d)** no

37. **(a)**



(b) 6 weeks
(c) 22 weeks after its release

39. **(a)** yes, $x = -1$ **(b)** no; domain is $x \geq 5$
(c) yes, $y = -58.5731$
(d) At 0°F, as the wind speed increases, there is a limiting wind chill of about −58.6°F. This is meaningful because at high wind speeds, additional wind probably has little noticeable effect.

41. **(a)** $P = C$ **(b)** C **(c)** $P' = 0$ **(d)** 0

43. **(a)** 0
(b) As the years past 1800 increase, the percentage of workers in farm occupations approaches 0.
(c) no

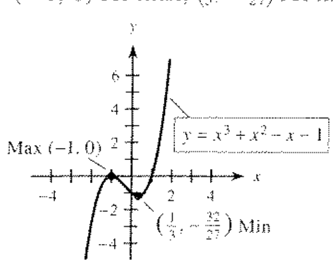
45. **(a)** No. Barometric pressure can drop off the scale (as shown), but it cannot decrease without bound. In fact, it must always be positive.
(b) See your library with regard to the "Storm of the Century" in March 1993.

Chapter 10 Review Exercises *(page 802)*

1. (0, 0) max **2.** (2, −9) min **3.** HPI (1, 0)

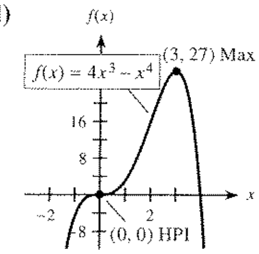
4. $\left(1, \frac{3}{2}\right)$ max, $\left(-1, -\frac{3}{2}\right)$ min

5. **(a)** $\frac{1}{3}, -1$ **(b)** (−1, 0) rel max, $\left(\frac{1}{3}, -\frac{32}{27}\right)$ rel min
(c) none **(d)**

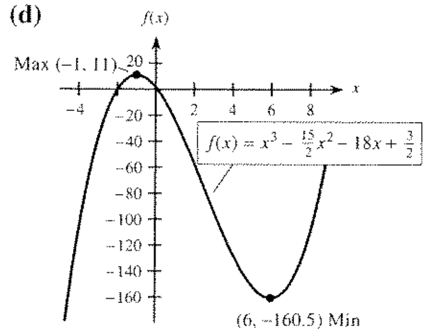


6. **(a)** 3, 0 **(b)** (3, 27) max **(c)** (0, 0)

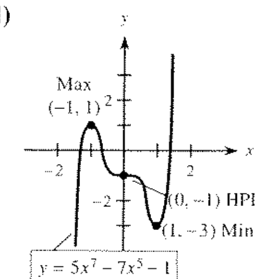
(d)



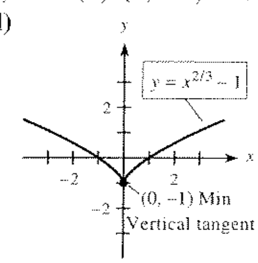
7. **(a)** −1, 6
(b) (−1, 11) rel max, (6, −160.5) rel min
(c) none **(d)**



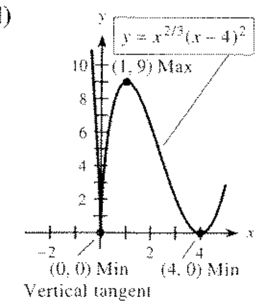
8. **(a)** 0, ±1 **(b)** (−1, 1) rel max, (1, −3) rel min
(c) (0, −1) **(d)**



9. **(a)** 0 **(b)** (0, −1) min **(c)** none
(d)



10. **(a)** 0, 1, 4
(b) (0, 0) rel min, (1, 9) rel max, (4, 0) rel min
(c) none **(d)**

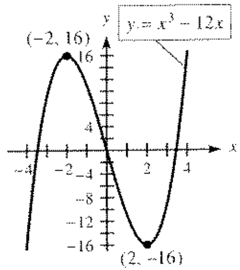


11. concave up

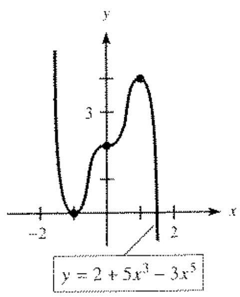
12. concave up when $x < -1$ and $x > 2$; concave down when $-1 < x < 2$; points of inflection at $(-1, -3)$ and $(2, -42)$

13. $(-1, 15)$ rel max; $(3, -17)$ rel min; point of inflection $(1, -1)$

14. $(-2, 16)$ rel max; $(2, -16)$ rel min; point of inflection $(0, 0)$



15. $(1, 4)$ rel max; $(-1, 0)$ rel min; points of inflection: $\left(\frac{1}{\sqrt{2}}, 2 + \frac{7}{4\sqrt{2}}\right)$, $(0, 2)$, and $\left(-\frac{1}{\sqrt{2}}, 2 - \frac{7}{4\sqrt{2}}\right)$



16. **(a)** $(0, 0)$ absolute min; $(140, 19{,}600)$ absolute max
(b) $(0, 0)$ absolute min; $(100, 18{,}000)$ absolute max

17. **(a)** $(50, 233{,}333)$ absolute max; $(0, 0)$ absolute min
(b) $(64, 248{,}491)$ absolute max; $(0, 0)$ absolute min

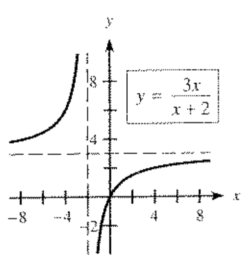
18. **(a)** $x = 1$ **(b)** $y = 0$ **(c)** 0 **(d)** 0

19. **(a)** $x = -1$ **(b)** $y = \frac{1}{2}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{2}$

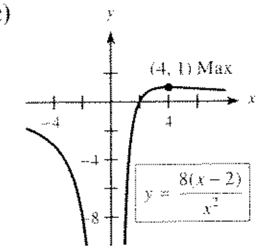
20. HA: $y = \frac{3}{2}$; VA: $x = 2$

21. HA: $y = -1$; VA: $x = 1, x = -1$

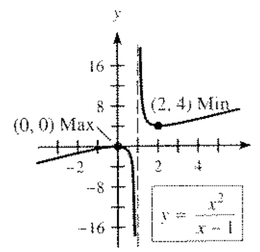
22. **(a)** HA: $y = 3$; VA: $x = -2$
(b) no max or min **(c)**



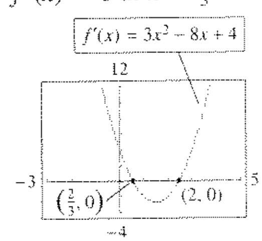
23. **(a)** HA: $y = 0$; VA: $x = 0$
(b) $(4, 1)$ max **(c)**



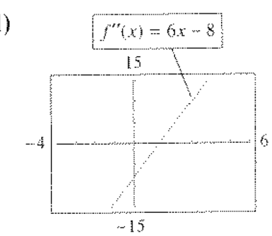
24. **(a)** HA: none; VA: $x = 1$
(b) $(0, 0)$ rel max; $(2, 4)$ rel min
(c)



25. **(a)** $f'(x) > 0$ for $x < \frac{2}{3}$ (approximately) and $x > 2$
$f'(x) < 0$ for about $\frac{2}{3} < x < 2$
$f'(x) = 0$ about $x = \frac{2}{3}$ and $x = 2$
(b) $f''(x) > 0$ for $x > \frac{4}{3}$
$f''(x) < 0$ for $x < \frac{4}{3}$
$f''(x) = 0$ at $x = \frac{4}{3}$
(c)

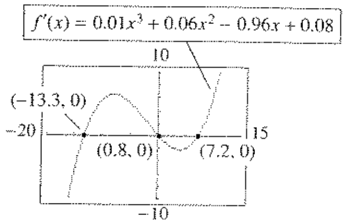


(d)

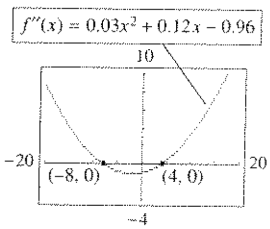


26. **(a)** $f'(x) > 0$ for about $-13 < x < 0$ and $x > 7$
$f'(x) < 0$ for about $x < -13$ and $0 < x < 7$
$f'(x) = 0$ for about $x = 0, x = -13, x = 7$
(b) $f''(x) > 0$ for about $x < -8$ and $x > 4$
$f''(x) < 0$ for about $-8 < x < 4$
$f''(x) = 0$ for about $x = -8$ and $x = 4$

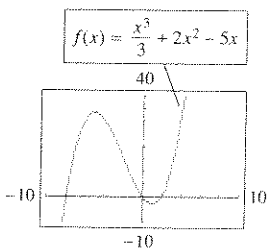
(c)



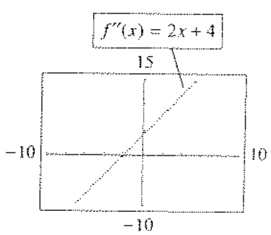
(d)



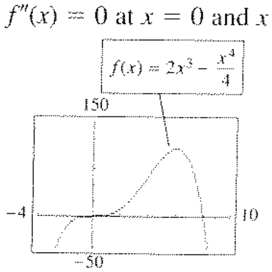
27. **(a)** $f(x)$ increasing for $x < -5$ and $x > 1$
$f(x)$ decreasing for $-5 < x < 1$
$f(x)$ has rel max at $x = -5$, rel min at $x = 1$
(b) $f''(x) > 0$ for $x > -2$ (where $f'(x)$ increases)
$f''(x) < 0$ for $x < -2$ (where $f'(x)$ decreases)
$f''(x) = 0$ for $x = -2$
(c)



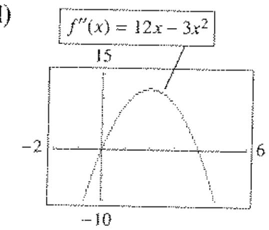
(d)



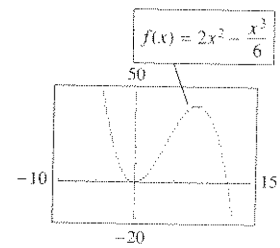
28. **(a)** $f(x)$ increasing for $x < 6$, $x \neq 0$
$f(x)$ decreasing for $x > 6$
$f(x)$ has max at $x = 6$, point of inflection at $x = 0$
(b) $f''(x) > 0$ for $0 < x < 4$
$f''(x) < 0$ for $x < 0$ and $x > 4$
$f''(x) = 0$ at $x = 0$ and $x = 4$
(c)



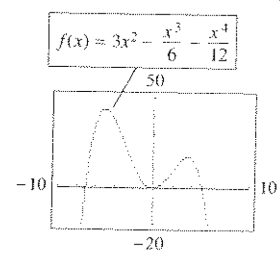
(d)



29. **(a)** $f(x)$ is concave up for $x < 4$
$f(x)$ is concave down for $x > 4$
$f(x)$ has point of inflection at $x = 4$
(b) $f''(x) = 4 - x$



30. **(a)** $f(x)$ is concave up for $-3 < x < 2$
$f(x)$ is concave down for $x < -3$ and $x > 2$
$f(x)$ has points of inflection at $x = -3$ and $x = 2$
(b) $f''(x) = 6 - x - x^2$



31. $x = 5$ units, $\overline{C} = \$45$ per unit
32. **(a)** $x = 1600$ units, $R = \$25{,}600$
(b) $x = 1600$ units, $R = \$25{,}600$
33. $P = \$54{,}000$ at $x = 100$ units **34.** $x = 3$ units
35. $x = 150$ units **36.** $x = 7$ units
37. $x = 500$ units, when $\overline{MP} = 0$ and changes from positive to negative.
38. 30 hours
39. **(a)** $I = 60$. The point of diminishing returns is located at the point of inflection (where bending changes).
(b) $m = f(I)/I =$ the average output
(c) The segment from (0, 0) to $y = f(I)$ has maximum slope when it is tangent to $y = f(I)$, close to $I = 70$.
40. \$260 per bike **41.** \$360 per bike
42. \$93,625 at 325 units
43. **(a)** 150 **(b)** \$650
44. \$208,490.67 at 64 units
45. $x = 1000$ mg **46.** 10:00 A.M.
47. 325 in 2005 **48.** 20 mi from A, 10 mi from B
49. 4 ft × 4 ft **50.** $8\frac{3}{4}$ in. × 10 in.
51. 500 mg **52.** $\frac{29}{18}$ lumens **53.** 24,000

13. $y' = \dfrac{-y}{2(x-1)}$ **15.** $\dfrac{dp}{dq} = \dfrac{p^2}{4-2pq}$

17. $\dfrac{dy}{dx} = \dfrac{x(3x^3-2)}{3y^2(1+y^2)}$ **19.** $\dfrac{dy}{dx} = \dfrac{4x^3+6x^2y^2-1}{-4x^3y-3y^2}$

21. $\dfrac{(4x^3+9x^2y^2-8x-12y)}{(18y+12x-6x^3y+10y^4)}$ **23.** undefined

25. 1 **27.** $y = \frac{1}{2}x + 1$ **29.** $y = 4x + 5$

31. $y' = \dfrac{1}{2xy}$ **33.** $y' = \dfrac{-y}{2x \ln x}$ **35.** -4

37. $-1/x$ **39.** $-y/x$

41. $ye^y/(1-e^x)$ **43.** 0 **45.** $y = -\frac{1}{3}x + 1$

47. horizontal: $(2, \sqrt{2}), (2, -\sqrt{2})$;
vertical: $(2 + 2\sqrt{2}, 0), (2 - 2\sqrt{2}, 0)$

49. **(c)** yes, because $x^2 + y^2 = 4$ **51.** $1/(2x\sqrt{x})$

53. max at $(0, 3)$; min at $(0, -3)$

55. $\frac{1}{2}$ so \$500 of sales from \$1000 of advertising

57. $-\frac{243}{128}$ hours of skilled labor per hour of unskilled labor

59. At $p = \$80$, $q = 49$ and $dq/dp = -\frac{5}{16}$, which means that if the price is increased to \$81, quantity demanded will decrease by approximately $\frac{5}{16}$ unit.

61. $-0.000436y$ **63.** $\dfrac{dh}{dt} = -\dfrac{3}{44} - \dfrac{h}{12}$

Exercise 11.4 *(page 846)*

1. 36 **3.** $\frac{1}{8}$ **5.** $-\frac{24}{5}$ **7.** $\frac{7}{6}$

9. -5 if $z = 5$, -10 if $z = -5$

11. -80 units/sec **13.** 12π ft²/min

15. $\frac{16}{27}$ in./sec **17.** \$1798/day **19.** \$0.42/day

21. 430 units/month **23.** 36π mm³/month

25. $\dfrac{\frac{dW}{dt}}{W} = 3\left(\dfrac{\frac{dL}{dt}}{L}\right)$ **27.** $\dfrac{\frac{dC}{dt}}{C} = 1.54\left(\dfrac{\frac{dW}{dt}}{W}\right)$

29. $\dfrac{1}{4\pi}$ micrometers/day **31.** $1/(20\pi)$ in./min

33. -0.75 ft/sec **35.** $-120\sqrt{6}$

37. approaching at 61.18 mph **39.** $\frac{1}{25}$ ft/hr

Exercise 11.5 *(page 856)*

1. **(a)** 1 **(b)** no change **3.** **(a)** 84
(b) Revenue will decrease.

5. **(a)** $\frac{100}{99}$ **(b)** elastic **(c)** decrease

7. **(a)** 0.81 **(b)** inelastic **(c)** increase

9. **(a)** $\eta = 11.1$ (approximately) **(b)** elastic

11. **(a)** $\eta = \dfrac{375 - 3q}{q}$
(b) unitary: $q = 93.75$; inelastic: $q > 93.75$; elastic: $q < 93.75$
(c) As q increases over $0 < q < 93.75$, p decreases, so elastic demand means R increases. Similarly, R decreases for $q > 93.75$.
(d) Maximum for R when $q = 93.75$; yes.

13. \$12/item **15.** $t = \$350$ **17.** \$115/item

19. \$483 per item; \$40,100 **21.** \$1100/item

Chapter 11 Review Exercises *(page 858)*

1. $(6x-1)e^{3x^2-x}$ **2.** $2x$ **3.** $1/q - \left[\dfrac{2q}{(q^2-1)}\right]$

4. $dy/dx = e^{x^2}(2x^2+1)$

5. $f'(x) = 10e^{2x} + 4e^{-0.1x}$

6. $g'(x) = 18e^{3x+1}(2e^{3x+1} - 5)^2$

7. $\dfrac{dy}{dx} = \dfrac{12x^3 + 14x}{3x^4 + 7x^2 - 12}$

8. $\dfrac{ds}{dx} = \dfrac{9x^{11} - 6x^3}{x^{12} - 2x^4 + 5}$ **9.** $dy/dx = 3^{3x-3}\ln 3$

10. $dy/dx = \dfrac{1}{\ln 8}\left(\dfrac{10}{x}\right)$ **11.** $y' = \dfrac{1 - \ln x}{x^2}$

12. $dy/dx = -2e^{-x}/(1 - e^{-x})^2$

13. $y = 12ex - 8e$, or $y = 32.62x - 21.75$

14. $y = x - 1$ **15.** $y' = \dfrac{y}{x(5 - \ln x)}$

16. $dy/dx = ye^{xy}/(1 - xe^{xy})$ **17.** $dy/dx = 2/y$

18. $\dfrac{dy}{dx} = \dfrac{2(x+1)}{3(1-2y)}$ **19.** $y' = \dfrac{6x(1+xy^2)}{y(5y^3 - 4x^3)}$

20. $d^2y/dx^2 = -(x^2+y^2)/y^3 = -1/y^3$ **21.** 5/9

22. $(-2, \pm\sqrt{\frac{2}{3}})$ **23.** 3/4 **24.** 11 square units/min

25. **(a)** $y'(t) = \dfrac{2.62196}{t}$
(b) $y(50) \approx 6.343$ is the predicted number of hectares of deforestation in 2000. $y'(50) \approx 0.05244$ hectares per year is the predicted rate of deforestation in 2000.

26. 135.3 **27.** **(a)** 152.5 **(b)** 1.13 times faster

28. **(a)** $-0.00001438A_0$ **(b)** $-0.00002876A_0$
(c) less

29. $\$1200e \approx \$3261.94$ per unit

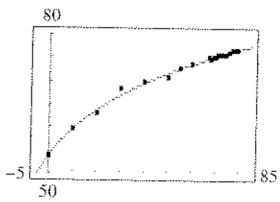
30. $-\$603.48$ per year **31.** $1/(25\pi)$ mm/min

32. $\frac{48}{25}$ ft/min **33.** $\dfrac{dS/dt}{S} = \dfrac{1}{3}\left(\dfrac{dA/dt}{A}\right)$ **34.** yes

35. $t = \$1446.67$ **36.** \$880

37. **(a)** 1 **(b)** no change **38.** $\frac{25}{12}$, elastic **39.** 1

(b) Graph of $I(t)$ with data points



(c) The model is an excellent fit.

55. (a) Yes, the rate is a growth exponential, so it is positive and increasing (hence the tax per capita is increasing).

(b) $T = 246.47\,e^{0.06891t} - 4.369$

(c) $T(55) = 10{,}904$ and $T'(55) = 751.69$ mean that the model predicts that in 2005, federal tax per capita will be \$10,904 and will be changing at the rate of \$751.69 per year.

Exercise 12.4 *(page 902)*

1. $C(x) = x^2 + 100x + 200$

3. $C(x) = 2x^2 + 2x + 80$ **5.** \$3750

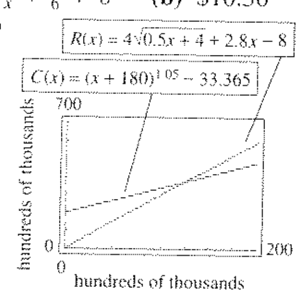
7. (a) $x = 3$ units is optimal level

(b) $P(x) = -4x^2 + 24x - 200$ **(c)** loss of \$164

9. (a) \$3120 **(b)** 896

11. (a) $\overline{C}(x) = \frac{6}{x} + \frac{x}{6} + 8$ **(b)** \$10.50

13. (a) and **(b)**



(c) Maximum profit is \$114.743 thousand at $x = 200$ thousand units.

15. $C(y) = 0.80y + 7$

17. $C(y) = 0.3y + 0.4\sqrt{y} + 8$

19. $C = 2\sqrt{y + 1} + 0.4y + 4$

21. $C = 0.7y + 0.5e^{-2y} + 5.15$

23. $C = 0.85y + 5.15$

25. $C = 0.8y + \dfrac{2\sqrt{3y + 7}}{3} + 4.24$

Exercise 12.5 *(page 911)*

1. $4y - 2xy' = 4x^2 - 2x(2x) = 0$ ✓

3. $2y\,dx - x\,dy = 2(3x^2 + 1)\,dx - x(6x\,dx) = 2\,dx$ ✓

5. $y = \frac{1}{2}e^{x^2+1} + C$ **7.** $y^2 = 2x^2 + C$

9. $y^3 = x^2 - x + C$ **11.** $y = e^{x-3} - e^{-3} + 2$

13. $y = \ln|x| - \dfrac{x^2}{2} + \dfrac{1}{2}$ **15.** $\dfrac{y^2}{2} = \dfrac{x^3}{3} + C$

17. $\dfrac{1}{2x^2} + \dfrac{y^2}{2} = C$ **19.** $\dfrac{1}{x} + y + \dfrac{y^3}{3} = C$

21. $\dfrac{1}{y} + \ln|x| = C$ **23.** $x^2 - y^2 = C$

25. $y = C(x + 1)$ **27.** $y - \ln|y + 1| = -\frac{1}{2}e^{-2x} + C$

29. $3y^4 = 4x^3 - 1$

31. $2y = 3x + 4xy$ or $y = \dfrac{3x}{2 - 4x}$

33. $e^{2y} = x^2 - \dfrac{2}{x} + 2$ **35.** $y^2 + 1 = 5x$

37. $y = Cx^k$

39. (a) $x = 10{,}000e^{0.06t}$ **(b)** \$10,618.37; \$13,498.59

(c) 11.55 years

41. ≈ 8.4 hours **43.** ≈ 23,100 years

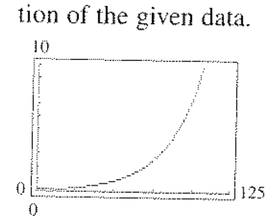
45. $x = 6(1 - e^{-0.05t})$ **47.** $x = 20 - 10e^{-0.025t}$

49. $y = \dfrac{32}{(p + 8)^{2/5}}$ **51.** $V = 1.86e^{2 - 2e^{-0.1t}}$

53. $V = \dfrac{k^3t^3}{27}$ **55.** $t \approx 4.5$ hours

57. (a) $E(t) = 0.1e^{0.043t}$

b) The graph is a similar, but smooth, representation of the given data.



59. (a) $P = 50{,}000e^{-0.05t}$ **(b)** \$11,156.50

Chapter 12 Review Exercises *(page 916)*

1. $\frac{1}{7}x^7 + C$ **2.** $\frac{2}{3}x^{3/2} + C$

3. $\frac{1}{4}x^4 - x^3 + 2x^2 + 5x + C$

4. $\frac{1}{5}x^5 - \frac{2}{3}x^3 + x + C$

5. $\frac{1}{6}(x^2 - 1)^3 + C$ **6.** $\frac{1}{18}(x^3 - 3x^2)^6 + C$

7. $\frac{1}{8}x^8 + \frac{8}{5}x^5 + 8x^2 + C$ **8.** $\frac{1}{21}(x^3 + 4)^7 + C$

9. $\frac{1}{3}\ln|x^3 + 1| + C$ **10.** $\dfrac{-1}{3(x^3 + 1)} + C$

11. $\frac{1}{2}(x^3 - 4)^{2/3} + C$ **12.** $\frac{1}{3}\ln|x^3 - 4| + C$

13. $\dfrac{1}{2}x^2 - \dfrac{1}{x} + C$ **14.** $\frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x - \ln|x - 1| + C$

15. $\frac{1}{3}e^{y^3} + C$ **16.** $x^3/3 - x^2 + x + C$

17. $\frac{1}{2}\ln|2x^3 - 7| + C$ **18.** $\dfrac{-5}{4e^{4x}} + C$

19. $x^4/4 - e^{3x}/3 + C$ **20.** $\frac{1}{2}e^{x^2+1} + C$

21. $\dfrac{-3}{40(5x^8 + 7)^2} + C$ **22.** $-\frac{7}{2}\sqrt{1 - x^4} + C$

23. $\frac{1}{4}e^{2x} - e^{-2x} + C$ 24. $x^2/2 + 1/(x+1) + C$
25. (a) $\frac{1}{10}(x^2-1)^5 + C$ (b) $\frac{1}{22}(x^2-1)^{11} + C$
(c) $\frac{3}{16}(x^2-1)^8 + C$ (d) $\frac{3}{2}(x^2-1)^{1/3} + C$
26. (a) $\ln|x^2-1| + C$ (b) $\frac{-1}{x^2-1} + C$
(c) $3\sqrt{x^2-1} + C$ (d) $\frac{3}{2}\ln|x^2-1| + C$
27. $y = C - 92e^{-0.05t}$
28. $y = 64x + 38x^2 - 12x^3 + C$
29. $(y-3)^2 = 4x^2 + C$ 30. $(y+1)^2 = 2\ln|t| + C$
31. $e^y = \frac{x^2}{2} + C$ 32. $y = Ct^4$
33. $3(y+1)^2 = 2x^3 + 75$
34. $x^2 = y + y^2 + 4$ 35. \$96 36. 472
37. $400[1 - 5/(t+5) + 25/(t+5)^2]$
38. $p = 1990.099 - 100{,}000/(t+100)$
39. (a) $y = -60e^{-0.04t} + 60$ (b) 23%
40. $R = 800\ln(x+1)$
41. (a) \$1000 (b) $C(x) = 3x^2 + 4x + 1000$
42. 80 units, \$440 43. $C = \sqrt{2y+16} + 0.6y + 4.5$
44. $C = 0.8y - 0.05e^{-2y} + 7.85$ 45. $W = CL^3$
46. $\approx$10.7 million years 47. $x = 360(1 - e^{-t/30})$
48. $x = 600 - 500e^{-0.01t}$; $\approx$ 161 min

Chapter 12 Test (page 918)

1. $2x^3 + 4x^2 - 7x + C$ 2. $4x + \frac{2}{3}x\sqrt{x} + \frac{1}{x} + C$
3. $\frac{(4x^3-7)^{10}}{24} + C$ 4. $\frac{(3x^2-6x+1)^{10}}{30} + C$
5. $\frac{\ln|2x^4-5|}{8} + C$ 6. $-10{,}000e^{-0.01x} + C$
7. $\frac{5}{8}e^{2y^4-1} + C$ 8. $e^x + 5\ln|x| - x + C$
9. $\frac{x^2}{2} - x + \ln|x+1| + C$ 10. $6x^2 - 1 + 5e^x$
11. $y = x^4 + x^3 + 4$ 12. $y = \frac{1}{4}e^{4x} + \frac{7}{4}$
13. $y = \frac{4}{C - x^4}$ 14. 157,498
15. $P(x) = 450x - 2x^2 - 300$
16. $C(y) = 0.78y + \sqrt{0.5y+1} + 5.6$
17. 332.3 days

Exercise 13.1 (page 931)

1. 7 square units 3. 22 square units
5. 3 square units 7. 30 square units
9. $S_L(10) = 4.08$; $S_R(10) = 5.28$
11. Both equal 14/3.
13. It would lie between $S_L(10)$ and $S_R(10)$. It would equal 14/3.
15. 3 17. 42 19. −5 21. 180 23. 11,315
25. $3 - \frac{3(n+1)}{n} + \frac{(n+1)(2n+1)}{2n^2} = \frac{2n^2-3n+1}{2n^2}$
27. (a) $S = (n-1)/2n$ (b) 9/20 (c) 99/200
(d) 999/2000 (e) $\frac{1}{2}$
29. (a) $S = \frac{(n+1)(2n+1)}{6n^2}$
(b) $77/200 = 0.385$ (c) $6767/20{,}000 \approx 0.3384$
(d) $667{,}667/2{,}000{,}000 \approx 0.3338$ (e) $\frac{1}{3}$ 31. $\frac{20}{3}$
33. \$234 billion
35. There are approximately 90 squares under the curve, each representing 1 second by
$10 \text{ mph} = 10 \times \frac{1 \text{ hr}}{3600} \times \frac{1 \text{ mile}}{\text{hour}} = \frac{1}{360}\text{mile}.$
The area under the curve is approximately $90(\frac{1}{360}\text{ mile}) = \frac{1}{4}$ mile.
37. 1550 square feet

Exercise 13.2 (page 942)

1. 18 3. 2 5. 60 7. $12\sqrt[3]{25}$ 9. 98
11. $\frac{7}{3}$ 13. 12,960 15. 0 17. $8\sqrt{3} - \frac{7}{3}\sqrt{7}$
19. 2 21. $e^3/3 - 1/3$ 23. 4 25. 0
27. (a) $\frac{1}{6}\ln(112/31) \approx 0.2140853$ (b) 0.2140853
29. (a) $\frac{3}{2} + 3\ln 2 \approx 3.5794415$ (b) 3.5794415
31. (a) A, C (b) B
33. $\int_0^4 (2x - \frac{1}{2}x^2)\,dx$ (b) 16/3
35. (a) $\int_{-1}^0 (x^3+1)\,dx$ (b) 3/4
37. $\frac{1}{6}$ 39. $\frac{1}{2}(e^9 - e)$
41. same absolute values, opposite signs
43. 6 45. (a) \$450,000 (b) \$450,000
47. (a) \$7007 (b) \$19,649
49. \$20,405.39 51. 0.04 cm^3
53. 1222 (approximately)
55. (a) $y = 0.005963x^{0.701215}$
(b) 0.15 mile (approximately)

Exercise 13.3 (page 953)

1. (a) $\int_0^2 (4 - x^2)\,dx$ (b) $\frac{16}{3}$
3. (a) $\int_1^8 [\sqrt[3]{x} - (2-x)]\,dx$ (b) 28.75
5. (a) $\int_1^2 [(4-x^2) - (\frac{1}{4}x^3 - 2)]\,dx$ (b) 131/48
7. (a) $(-1, 1), (2, 4)$ (b) $\int_{-1}^2 [(x+2) - x^2]\,dx$
(c) 9/2
9. (a) $(0, 0), (\frac{5}{2}, -\frac{15}{4})$
(b) $\int_0^{5/2} [(x - x^2) - (x^2 - 4x)]\,dx$ (c) $\frac{125}{24}$
11. (a) $(-2, -4), (0, 0), (2, 4)$
(b) $\int_{-2}^0 [(x^3 - 2x) - 2x]\,dx + \int_0^2 [2x - (x^3 - 2x)]\,dx$
(c) 8
13. $\frac{28}{3}$ 15. $\frac{1}{4}$ 17. $\frac{16}{3}$ 19. $\frac{1}{3}$ 21. $\frac{37}{12}$
23. $4 - 3\ln 3$ 25. $\frac{8}{3}$ 27. 6 29. 0 31. $-\frac{4}{9}$

33. $11.8\overline{3}$
35. average profit $= \dfrac{1}{x_1 - x_0}\displaystyle\int_{x_0}^{x_1} (R(x) - C(x))\,dx$
37. **(a)** \$1402 per unit **(b)** \$535,333.33
39. **(a)** 102.5 units **(b)** 100 units
41. 7.40% **43.** 147 milligrams
45. 1980, 0.351; 1990, 0.377. The difference in incomes widened.
47. whites, 0.391; blacks, 0.439. In 1996, income was more nearly equally distributed among whites.

Exercise 13.4 *(page 966)*

1. \$120,000 **3.** \$346,664 (nearest dollar)
5. \$506,000 (nearest thousand)
7. \$18,660 (nearest dollar)
9. \$82,155 (nearest dollar)
11. PV = \$265,781 (nearest dollar), FV = \$377,161 (nearest dollar)
13. PV = \$190,519 (nearest dollar), FV = \$347,148 (nearest dollar)
15. Gift Shoppe, \$151,024; Video Palace, \$141,093. The gift shop is a better buy.
17. \$83.33 **19.** \$161.89 **21.** (5, 56); \$83.33
23. \$11.50 **25.** \$204.17 **27.** \$2766.67
29. \$17,839.58 **31.** \$133.33 **33.** \$2.50
35. \$103.35

Exercise 13.5 *(page 973)*

1. $\frac{1}{8}\ln|(4 + x)/(4 - x)| + C$
3. $\frac{1}{3}\ln[(3 + \sqrt{10})/2]$ **5.** $w(\ln w - 1) + C$
7. $\frac{1}{3} + \frac{1}{4}\ln(\frac{3}{7})$ **9.** $3^x \log_3 e + C$ or $3^x/\ln 3 + C$
11. $\frac{1}{2}[7\sqrt{24} - 25\ln(7 + \sqrt{24}) + 25\ln 5]$
13. $\dfrac{(6w - 5)(4w + 5)^{3/2}}{60} + C$ **15.** $\frac{1}{2}(5^{x^2})\log_5 e + C$
17. $\frac{1}{3}(13^{3/2} - 8)$ **19.** $-\frac{5}{2}\ln\left|\dfrac{2 + \sqrt{4 - 9x^2}}{3x}\right| + C$
21. $\frac{1}{3}\ln|3x + \sqrt{9x^2 - 4}| + C$ **23.** $\frac{1}{8}\ln(\frac{9}{5})$
25. $\frac{1}{3}\ln|3x + 1 + \sqrt{(3x + 1)^2 + 1}| + C$
27. $\frac{1}{4}[10\sqrt{109} - \sqrt{10} + 9\ln(10 + \sqrt{109}) - 9\ln(1 + \sqrt{10})]$
29. $-\frac{1}{6}\ln|7 - 3x^2| + C$
31. $\frac{1}{2}\ln|2x + \sqrt{4x^2 + 7}| + C$
33. $2(e^{\sqrt{2}} - e) \approx 2.7899$
35. $\frac{1}{32}[\ln(9/5) - 4/9] \approx .004479$ **37.** \$3391.10
39. **(a)** $C = \frac{1}{2}x\sqrt{x^2 + 9} + \frac{9}{2}\ln\left|\dfrac{x + \sqrt{x^2 + 9}}{3}\right| + 300$
(b) \$314.94
41. \$3882.9 thousand

Exercise 13.6 *(page 979)*

1. $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$ **3.** $\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$
5. $\dfrac{104\sqrt{2}}{15}$ **7.** $-(1 + \ln x)/x + C$ **9.** 1
11. $\dfrac{x^2}{2}\ln(2x - 3) - \dfrac{1}{4}x^2 - \dfrac{3}{4}x - \dfrac{9}{8}\ln(2x - 3) + C$
13. $\frac{1}{5}(q^2 - 3)^{3/2}(q^2 + 2) + C$ **15.** 282.4
17. $-e^{-x}(x^2 + 2x + 2) + C$ **19.** $(3e^4 + 1)/2$
21. $\frac{1}{4}x^4\ln^2 x - \frac{1}{8}x^4\ln x + \frac{1}{32}x^4 + C$
23. $\frac{2}{15}(e^x + 1)^{3/2}(3e^x - 2) + C$ **25.** II; $\frac{1}{2}e^{x^2} + C$
27. IV; $\frac{2}{3}(e^x + 1)^{3/2} + C$ **29.** I; $-5e^{-4} + 1$
31. \$2794.46 **33.** \$34,836.73 **35.** 0.264

Exercise 13.7 *(page 987)*

1. 1/5 **3.** 2 **5.** $1/e$ **7.** diverges **9.** diverges
11. 10 **13.** diverges **15.** diverges **17.** diverges
19. 0 **21.** 0.5 **23.** $1/(2e)$ **25.** $\frac{3}{2}$
27. $\int_{-\infty}^{\infty} f(x)\,dx = 1$ **29.** $c = 1$ **31.** $c = \frac{1}{4}$
33. 20 **35.** area $= \frac{8}{3}$ **37.** $\int_0^{\infty} Ae^{-rt}\,dt = A/r$
39. \$2,400,000 **41.** \$700,000
43. **(a)** $500\left[\dfrac{e^{-0.03b} + 0.03b - 1}{0.0009}\right]$
(b) the amount approaches ∞
45. 0.147

Exercise 13.8 *(page 996)*

1. $h = \frac{1}{2}$; $x_0 = 0, x_1 = \frac{1}{2}, x_2 = 1, x_3 = \frac{3}{2}, x_4 = 2$
3. $h = \frac{1}{2}$; $x_0 = 1, x_1 = \frac{3}{2}, x_2 = 2, x_3 = \frac{5}{2}, x_4 = 3, x_5 = \frac{7}{2}, x_6 = 4$
5. $h = 1$; $x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3, x_5 = 4$
7. **(a)** 9.13 **(b)** 9.00 **(c)** 9 **(d)** Simpson's
9. **(a)** 0.51 **(b)** 0.50 **(c)** $\frac{1}{2}$ **(d)** Simpson's
11. **(a)** 5.27 **(b)** 5.30 **(c)** 5.33 **(d)** Simpson's
13. **(a)** 3.283 **(b)** 3.240 **15.** **(a)** 0.743 **(b)** 0.747
17. **(a)** 7.132 **(b)** 7.197 **19.** 7.8 **21.** 10.3
23. 119.58 (\$119,580) **25.** \$32,389.76
27. \$14,133.33 **29.** 1222.35 (1222 units)
31. 1586.67 ft²

Chapter 13 Review Exercises *(page 1001)*

1. 212 **2.** $\dfrac{3(n + 1)}{2n^2}$ **3.** $\frac{91}{72}$ **4.** 1 **5.** 1
6. 14 **7.** $\frac{248}{5}$ **8.** $-\frac{205}{4}$ **9.** $\frac{825}{4}$ **10.** $\frac{125}{3}$
11. -2 **12.** $\frac{1}{6}\ln 47 - \frac{1}{6}\ln 9$ **13.** $\frac{9}{2}$
14. $\ln 4 + \frac{14}{3}$ **15.** $\frac{26}{3}$ **16.** $\frac{1}{2}\ln 2$ **17.** $(1 - e^{-2})/2$
18. $(e - 1)/2$ **19.** 95/2 **20.** 36 **21.** $\frac{1}{4}$ **22.** $\frac{1}{2}$

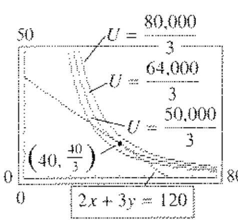
(c) $-400/(p_B + 4)^2$ units per dollar
(d) $400/(p_A + 4)^2$ units per dollar
(e) competitive

Exercise 14.4 (page 1044)

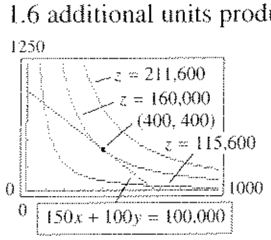
1. max (0, 0, 9) **3.** min(0, 0, 4) **5.** min(1, −2, 0)
7. saddle (1, −3, 8) **9.** saddle (0, 0, 0)
11. max(12, 24, 456) **13.** min(−8, 6, −52)
15. saddle (0, 0, 0); min(2, 2, −8)
17. $\hat{y} = 5.7x - 1.4$
19. $x = 5000$, $y = 128$ **21.** $x = \frac{20}{3}$, $y = \frac{10}{3}$
23. $x = 28$, $y = 100$ **25.** $x = 0$, $y = 10$
27. length = 100, width = 100, height = 50
29. $x = 3$, $y = 0$
31. (a) $y = 0.113x + 4.860$, x in years past 1970, y = median age of autos and trucks.
(b) 8.25
(c) The reasons include higher prices and better quality.
33. (a) $y = 2.097x + 59.102$, x in years past 1900, y in millions
(b) 258.3 million
(c) A quadratic model fits better. ($y = 0.010x^2 + 0.961x + 81.820$)

Exercise 14.5 (page 1053)

1. 18 at (3, 3) **3.** 35 at (3, 2) **5.** 32 at (4, 2)
7. −28 at $(3, \frac{5}{3})$ **9.** $\frac{1}{5}$ at $(-\frac{2}{5}, -\frac{1}{5})$
11. 3 at (1, 1, 1) **13.** 1 at (0, 1, 0)
15. $x = 4$, $y = 1$
17. $x = 40$, $y = \frac{40}{3}$



19. (a) $x = 400$, $y = 400$
(b) $-\lambda = 1.6$ means that each additional dollar spent on production results in approximately 1.6 additional units produced.
(c)



21. $x = 900$, $y = 300$
23. $x = \$10{,}003.33$, $y = \$19{,}996.67$
25. length = 100 cm, width = 100 cm, height = 50 cm

Chapter 14 Review Exercises (page 1056)

1. $\{(x, y)$: x and y are real numbers and $y \neq 2x\}$
2. $\{(x, y)$: x and y are real numbers with $y \geq 0$ and $(x, y) \neq (0, 0)\}$
3. −5 **4.** 896,000
5. $15x^2 + 6y$ **6.** $24y^3 - 42x^3y^2$
7. $z_x = 8xy^3 + 1/y$; $z_y = 12x^2y^2 - x/y^2$
8. $z_x = x/\sqrt{x^2 + 2y^2}$; $z_y = 2y/\sqrt{x^2 + 2y^2}$
9. $z_x = -2y/(xy + 1)^3$; $z_y = -2x/(xy + 1)^3$
10. $z_x = 2xy^3e^{x^2y^3}$; $z_y = 3x^2y^2e^{x^2y^3}$
11. $z_x = ye^{xy} + y/x$; $z_y = xe^{xy} + \ln x$
12. $z_x = y$; $z_y = x$ **13.** −8 **14.** 8
15. (a) $2y$ (b) 0 (c) $2x - 3$ (d) $2x - 3$
16. (a) $18xy^4 - 2/y^2$ (b) $36x^3y^2 - 6x^2/y^4$
(c) $36x^2y^3 + 4x/y^3$ (d) $36x^2y^3 + 4x/y^3$
17. (a) $2e^{y^2}$ (b) $4x^2y^2e^{y^2} + 2x^2e^{y^2}$
(c) $4xye^{y^2}$ (d) $4xye^{y^2}$
18. (a) $-y^2/(xy + 1)^2$ (b) $-x^2/(xy + 1)^2$
(c) $1/(xy + 1)^2$ (d) $1/(xy + 1)^2$
19. max(−8, 16, 208)
20. saddle(0, 0, 0); min(1, 1, −1)
21. 80 at (2, 8) **22.** 11,664 at (6, 3)
23. 3 units **24.** 20 units
25. (a) 62.8 km (approximately)
(b) If the surface temperature is 287 K, the Concorde's altitude is 17.1 km, and the vertical temperature gradient is 4.9 K/km, then the width of the sonic boom is about 63.3 km.
(c) $\frac{\partial f}{\partial h} \approx 1.87$; If the surface temperature is 293 K and the temperature gradient is 5 K/km, then a change of 1 km in the Concorde's altitude would result in a change in the width of the sonic boom of about 1.87 km.
(d) $\frac{\partial f}{\partial d} \approx -6.28$; If the surface temperature is 293 K and the Concorde's altitude is 16.8 km, then a change of 1 K/km in the temperature gradient would result in a change in the width of the sonic boom of about −6.28 km.
26. (a) 280 dollars per unit of x
(b) 2400/7 dollars per unit of y
27. $\partial Q/\partial K = 81.92$ means when capital expenditures increase by \$1000 (to \$626,000) and work-hours remain at 4096, output will change by about 8192 units; $\partial Q/\partial L = 37.5$ means that when labor hours change by 1 (to 4097) and capital expenditures remain at \$625,000, output will change by about 3750 units.
28. (a) −2 (b) −6 (c) complementary
29. competitive **30.** $x = 20$, $y = 40$