

Problem 1 (Fractions). Simplify:

$$\text{LCD} = 15$$

$$\left(\frac{\frac{3}{5} - \frac{2}{3}}{\frac{1}{3} + \frac{3}{5}} \right) \frac{15}{15}$$

$$= \frac{\frac{3}{5}(15) - \frac{2}{3}(15)}{\frac{1}{3}(15) + \frac{3}{5}(15)} = \frac{9 - 10}{5 + 9} = \left(\frac{-1}{14} \right)$$

Problem 2 (A Linear Equation). Solve the equation

$$4x - 3 = 10 - 2(x - 1)$$

$$4x - 3 = 10 - 2x + 2$$

$$\begin{array}{r} 4x - 3 = 12 - 2x \\ +2x \quad +3 \quad +3 \quad +2x \end{array}$$

$$6x = 15$$

$$x = \frac{15}{6} = \left(\frac{5}{2} \right)$$

Problem 3 (A Quadratic Equation). Find all solutions of the equation

$$x^2 - x - 20 = 0$$

$$(x - 5)(x + 4) = 0$$

$$x - 5 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = 5 \quad \quad \quad x = -4$$

$$x = 5, -4$$

Problem 4 (Another Quadratic Equation). Find all solutions of the equation

$$x^2 - 2x - 5 = 0$$

$$a = 1 \quad b = -2 \quad c = -5$$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 20}}{2} = \frac{2 \pm \sqrt{24}}{2}$$

$$x = \frac{2 \pm 2\sqrt{6}}{2} = \frac{2(1 \pm \sqrt{6})}{2}$$

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$$x = 1 \pm \sqrt{6}$$

Problem 5 (A rational equation). Find all solutions of

$$\text{LCD} = (x-2)(x-3)$$

$$\frac{8}{x-2} - \frac{5}{x-3} + 1 = 0$$

$$x \neq 2, 3$$

$$\frac{(x-2)(x-3)8}{(x-2)} - \frac{5(x-2)(x-3)}{(x-3)} + 1(x-2)(x-3) = 0(x-2)(x-3)$$

$$8(x-3) - 5(x-2) + (x-2)(x-3) = 0$$

$$8x - 24 - 5x + 10 + x^2 - 2x - 3x + 6 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4, -2$$

Problem 6 (Polynomials). Write the following polynomial expression in standard form. What is its degree and its leading coefficient?

$$(x^2 - 1)(x + 3) + 2x + 4$$

$$= x^3 + 3x^2 - x - 3 + 2x + 4$$

$$= x^3 + 3x^2 + x + 1$$

$$\text{degree} = 3$$

$$\text{leading coefficient} = 1$$

For the next three questions let

$$f(x) = \frac{x-1}{x^2-2x} = \frac{x-1}{x(x-2)}$$

Problem 7 (Domain). What is the natural domain of f ?

$$x \in \mathbb{R}, x \neq 0, 2$$

Problem 8 (Evaluate at a point). Find $f(5)$.

$$f(5) = \frac{5-1}{5^2-2(5)} = \frac{4}{25-10} = \frac{4}{15}$$

Problem 9 (Evaluate at an expression). Find $f(2x+1)$ and express it in the standard form of a rational expression.

$$\begin{aligned} f(2x+1) &= \frac{2x+1-x}{(2x+1)^2-2(2x+1)} \\ &= \frac{x+1}{4x^2+4x+1-4x-2} \\ &= \frac{x+1}{4x^2-1} \end{aligned}$$

Problem 10 (Radical Equations). Solve the equation

$$\sqrt{x+4} + \sqrt{x+11} = 7.$$

$$\left(\sqrt{x+11}\right)^2 = \left(7 - \sqrt{x+4}\right)^2$$

$$x+11 = 49 - 14\sqrt{x+4} + \underset{-x}{x+4}$$

$$11 = 53 - 14\sqrt{x+4}$$

$$\frac{-42}{-14} = \frac{-14\sqrt{x+4}}{-14}$$

$$3 = \left(\sqrt{x+4}\right)^2 \Rightarrow 9 = x+4$$

$$5 = x$$

Problem 11 (Linear System). Solve the system

$$4x - y = 1$$

$$2x + y = 0$$

Show all your work, don't just give the answer.

$$\begin{array}{r} 4x - y = 1 \\ + \quad 2x + y = 0 \\ \hline 6x = 1 \end{array}$$

$$x = \frac{1}{6}$$

$$4\left(\frac{1}{6}\right) - y = 1 \Rightarrow \frac{2}{3} - y = 1$$

$$\frac{-2}{3} \qquad \frac{-2}{3}$$

$$-y = \frac{1}{3} \Rightarrow y = -\frac{1}{3}$$

5

$$\left(\frac{1}{6}, -\frac{1}{3}\right)$$

Problem 12 (Another Linear System). Solve the linear system

$$\begin{array}{rcl} \begin{array}{l} (-2) \\ \downarrow \end{array} & \begin{array}{l} (-1) \\ \downarrow \end{array} & \begin{array}{l} x + y + z = 5 \\ x + 2y - z = 3 \\ 2x + 3y - z = 5 \end{array} \end{array}$$

Again, show all your work, don't just give the answer.

$$\begin{array}{l} (-1) \\ \downarrow \end{array} \begin{array}{l} x + y + z = 5 \\ y - 2z = -2 \\ y - 3z = -5 \end{array}$$

$$\begin{array}{l} x + y + z = 5 \\ y - 2z = -2 \\ -z = -3 \end{array}$$

$$z = 3$$

$$y - 2(3) = -2$$

$$y - 6 = -2$$

$$y = 4$$

$$x + 4 + 3 = 5$$

$$x + 7 = 5$$

$$x = -2$$

$$(-2, 4, 3)$$

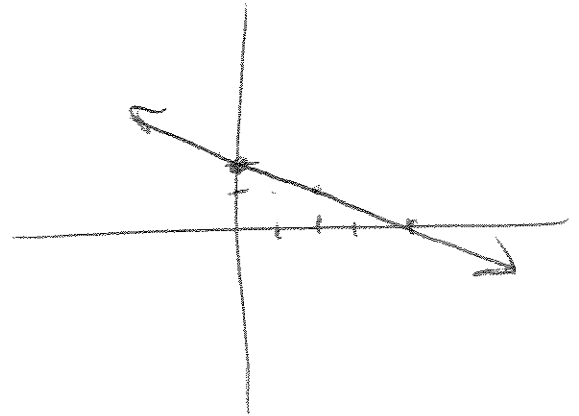
Problem 13 (Straight Lines). Find an equation of the line that passes through (2,1) and has slope $-1/2$. Draw its graph.

$$m = -\frac{1}{2} \quad \begin{matrix} x_1 & y_1 \\ (2, 1) \end{matrix}$$

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$y - 1 = -\frac{1}{2}x + 1$$

$$y = -\frac{1}{2}x + 2$$



Problem 14 (Distance). Find the distance between the points $(-1, 3)$ and $(2, 4)$.

$$d = \sqrt{(3-4)^2 + (-1-2)^2}$$

$$= \sqrt{(-1)^2 + (-3)^2}$$

$$= \sqrt{1 + 9} = \sqrt{10}$$

Problem 15 (Powers). Simplify (i.e., write with only positive exponents, such that x and y occur only once) the expression

$$\frac{(x^2y^{-3})^2}{(x^{-1}y)^{-3}} = \frac{x^4y^{-6}}{x^{-3}y^{-3}}$$

$$= \frac{x^4y^3}{x^3y^6} = \frac{x}{y^3}$$

Problem 16 (Radical Expressions). Simplify the expression

$$\left(\frac{x^{3/2}}{x^{1/6}}\right)^{2/5}$$

and write it as a power with a single exponent.

$$= \left(x^{\frac{3}{2} - \frac{1}{6}}\right)^{\frac{2}{5}}$$

$$= \left(x^{\frac{4}{3}}\right)^{\frac{2}{5}}$$

$$= x^{\frac{8}{15}}$$

$$\left(\frac{3}{3}\right)\frac{3}{2} - \frac{1}{6} = \frac{9}{6} - \frac{1}{6}$$

$$= \frac{8}{6}$$

$$= \frac{4}{3}$$

Problem 17 (Rational Expressions). Simplify the following expression

$$\text{LCD} = (x+1)(x-2)(x-3)$$

$$\frac{2}{x+1} + \frac{1}{x-2} - \frac{2}{x-3}$$

$$\begin{aligned} & \frac{2}{(x+1)} \left(\frac{(x-2)(x-3)}{(x-2)(x-3)} \right) + \frac{1}{(x-2)} \left(\frac{(x+1)(x-3)}{(x+1)(x-3)} \right) - \frac{2}{(x-3)} \left(\frac{(x+1)(x-2)}{(x+1)(x-2)} \right) \\ &= \frac{2(x^2 - 5x + 6) + x^2 - 2x - 3 - 2(x^2 - x - 3)}{(x+1)(x-2)(x-3)} \\ &= \frac{\cancel{2x^2} - 10x + 12 + x^2 - \cancel{2x} - 3 - \cancel{2x^2} + \cancel{2x} + 6}{(x+1)(x-2)(x-3)} = \frac{x^2 - 10x + 15}{(x+1)(x-2)(x-3)} \end{aligned}$$

Problem 18 (Inequality). Solve the following inequality

$$|2x - 1| + 2 \leq 8$$

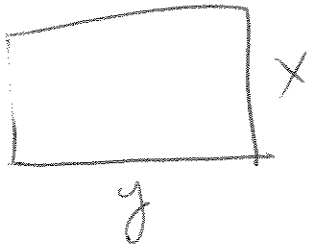
$$|2x - 1| \leq 6$$

$$\begin{array}{c} -6 \leq 2x - 1 \leq 6 \\ +1 \quad \quad +1 \quad \quad +1 \end{array}$$

$$\frac{-5}{2} \leq \frac{2x}{2} \leq \frac{7}{2}$$

$$\boxed{\frac{-5}{2} \leq x \leq \frac{7}{2}}$$

Problem 19 (Word Problem). You are at a store buying some fencing for your new garden. You would like it to have an area of 36 square meters, and to be in a shape of a rectangle. The store clerk cut 26 meters of chicken wire. How long will the sides of your garden be?



$$P = 2x + 2y = 26$$

$$A = xy = 36 \text{ m}^2$$

$$\textcircled{1} 2x + 2y = 26 \Rightarrow x + y = 13 \Rightarrow y = 13 - x \quad \text{substitute}$$

$$\textcircled{2} xy = 36 \Rightarrow x(13 - x) = 36$$

$$13x - x^2 = 36$$

$$\leftarrow 0 = x^2 - 13x + 36$$

if $x=4$, $y=9$
if $x=9$, $y=4$

$$(x-9)(x-4) = 0$$

$$x-9=0 \quad x-4=0$$

$$x=9 \quad x=4$$

Answer 9 m by 4 m

Problem 20 (Another Word Problem). You obtain 5 gallons of 14% vinegar solution by mixing suitable amounts of 5% vinegar solution and of 20% vinegar solution. How much of each type of vinegar solution do you use?

	% vinegar	# gallons liquid	# gallons vinegar
A	5%	x	$0.05x$
B	20%	$5-x$	$0.2(5-x)$
Mix	14%	5	$0.14(5)$

$$0.05x + 0.2(5-x) = 0.14(5)$$

$$100(0.05x + 1 - 0.2x) = (0.7)100$$

$$5x + 100 - 20x = 70$$

$$-15x = -30$$

$$x = 2$$

Answer

10

2 gallons of 5% solution
3 gallons of 20% solution