

Math1010 Equation Solving Strategies

- (1) Linear Equations —Simplify both sides of the equation, isolate the variable.

$$\begin{aligned}
 &3(x-2)+1=5x-7 \\
 &3x-6+1=5x-7 \\
 \text{Example: } &3x-5=5x-7 \\
 &-5=2x-7 \\
 &2=2x \\
 &1=x
 \end{aligned}$$

- (2) Quadratic Equations—

(a) If it has an x term in it, then move everything on one side of the equation with the other side zero. Try to factor it.

1. If it factors, set each factor equal to zero and solve.

2. If it doesn't factor, use the quadratic formula, i.e. For $ax^2+bx+c=0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

(b) If it doesn't have an x term in it, then isolate the x^2 and take the square root of both sides. (Remember to put the plus or minus sign on the answer.)

Examples:

$$\begin{aligned}
 &3x^2+13x=10 \\
 &3x^2+13x-10=0 \\
 &(3x-2)(x+5)=0 \\
 \text{(a) 1. } &3x-2=0 \text{ or } x+5=0 \\
 &3x=2 \text{ or } x=-5 \\
 &x=\frac{2}{3} \text{ or } x=-5
 \end{aligned}$$

$$\begin{aligned}
 &2x^2+x-5=0 \\
 &a=2, b=1, c=-5 \\
 &x = \frac{-1 \pm \sqrt{1-4(2)(-5)}}{2(2)} \\
 \text{(a) 2. } &x = \frac{-1 \pm \sqrt{1+40}}{4} \\
 &x = \frac{-1 \pm \sqrt{41}}{4}
 \end{aligned}$$

$$\begin{aligned}
 &2x^2-50=0 \\
 \text{(b) } &2x^2=50 \\
 &x^2=25 \\
 &x=\pm 5
 \end{aligned}$$

(Note: For quadratic equations, the quadratic formula method will ALWAYS work. So, if you'd like, you can always use that method.)

- (3) **Higher-Order Polynomial Equations**—Get everything on one side of the equation, with the other side zero. Factor and then set each factor equal to zero and solve. (This is basically a more intense version of the 2(a)1. type.)

$$10x^4 = 5x^3 + 15x^2$$

$$10x^4 - 5x^3 - 15x^2 = 0$$

$$5x^2(2x^2 - x - 3) = 0$$

Example: $5x^2(2x - 3)(x + 1) = 0$

$$5x^2 = 0 \text{ or } 2x - 3 = 0 \text{ or } x + 1 = 0$$

$$x = 0 \text{ or } x = \frac{3}{2} \text{ or } x = -1$$

- (4) **Rational Equations**—Find the least common denominator (LCD). Multiply both sides of the equation by the LCD which will get rid of the fractions. Then, it turns into either a linear or quadratic equation and you solve those using the previously stated methods.

$$\frac{1}{x+1} - \frac{2}{x+5} = \frac{x+2}{x+1}$$

$$LCD = (x+1)(x+5)$$

$$\frac{(x+1)(x+5)}{x+1} - \frac{2(x+1)(x+5)}{x+5} = \frac{(x+2)(x+1)(x+5)}{x+1}$$

Example:

$$x+5 - 2(x+1) = (x+2)(x+5)$$

$$x+5 - 2x - 2 = x^2 + 7x + 10$$

$$-x + 3 = x^2 + 7x + 10$$

$$0 = x^2 + 8x + 7$$

$$0 = (x+7)(x+1)$$

$$0 = x+7 \text{ or } 0 = x+1$$

$$x = -7 \text{ or } x = -1$$

(Note: You MUST check your answers for rational equations to make sure they don't give you a zero in the denominator of the original equation. If a solution makes the denominator go to zero, then you throw out that solution.)

(5) **Radical Equations**—Isolate the radical expression. If there are two radical expressions, isolate one of them (either one). Then raise both sides of the equation to the power that will “undo” the radical. So, if it's a square root, then you would square both sides. If it's a cube root, then cube both sides. If there is still a radical expression left in the equation, repeat this process, until there are no radical expressions left. Then, the equation will either be a linear or quadratic equation and you can solve appropriately.

$$\begin{aligned}
 &\sqrt{x-6}+3=\sqrt{x+9} \\
 &(\sqrt{x-6}+3)^2=(\sqrt{x+9})^2 \\
 &(\sqrt{x-6}+3)(\sqrt{x-6}+3)=x+9 \\
 &(x-6)+3\sqrt{x-6}+3\sqrt{x-6}+9=x+9 \\
 &x-6+9+6\sqrt{x-6}=x+9 \\
 \text{Example: } &x+3+6\sqrt{x-6}=x+9 \\
 &6\sqrt{x-6}=6 \\
 &\sqrt{x-6}=1 \\
 &(\sqrt{x-6})^2=1^2 \\
 &x-6=1 \\
 &x=7
 \end{aligned}$$

(Note: If your original equation has an even root in it, then you **MUST** check your solution by plugging it into the original equation to see if it really works. If it doesn't work, then throw it out.)