Math3105
Homework 8
Due next week in class

Homework Assigned:

1. Worksheet 76.
2. Worksheet 77.
3. Worksheets 78-79.
4. Create 4 iterations of the Koch snowflake.
5. Pages 29-30 (from "Fractals: A Toolkit of Dynamics Activities")

Here are some useful links for fractal information.

http://math.rice.edu/~lanius/fractals/dim.html

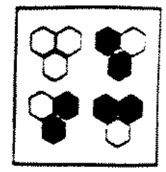
http://www.coolmath4kids.com/fractals.html

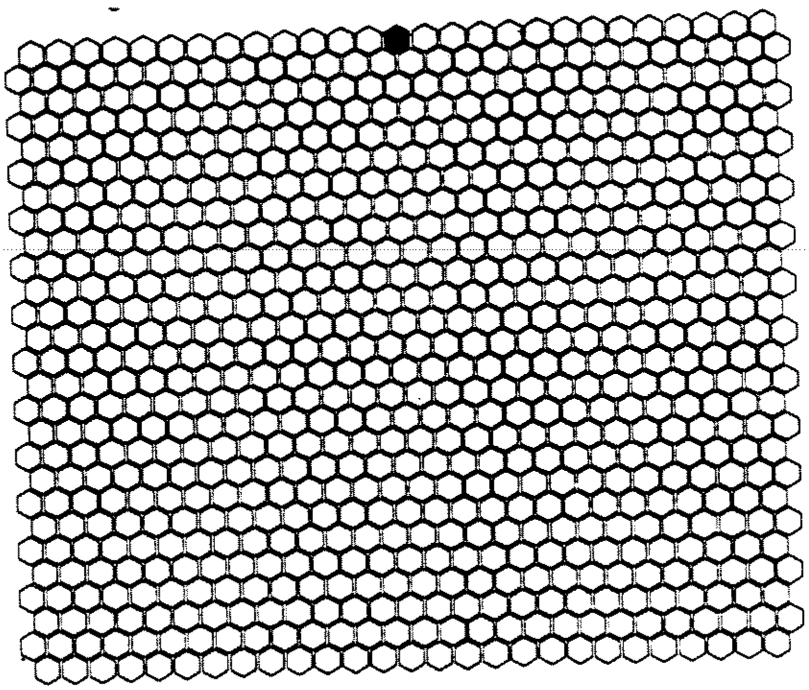
http://42explore.com/fractal.htm

Automata

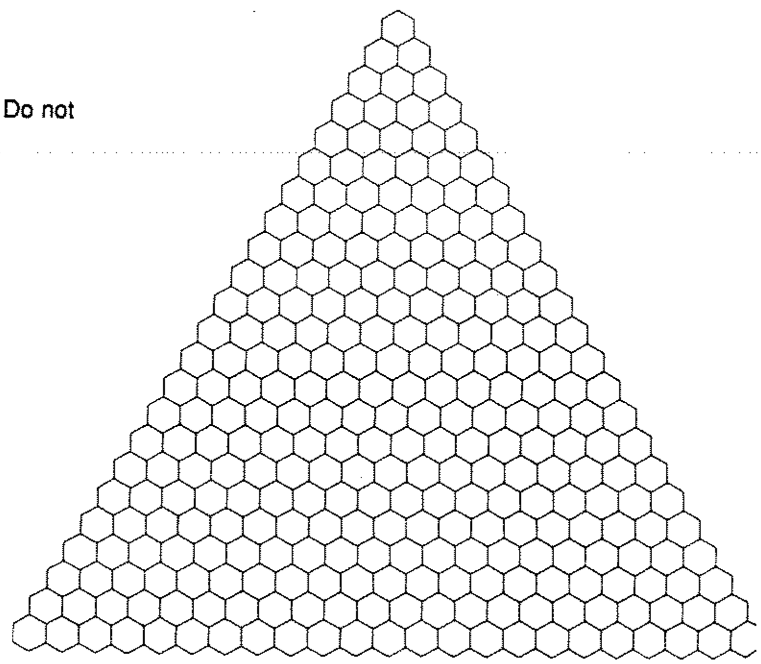
The rule for coloring the cells

If the two cells directly above are different in color, then shade in the cell so the color is black. If they are the same in color, leave the cell unshaded so the color is white





Shade only even blocks.



In rows 13, 14, and 15, enter only the letters E for *even* or O for *odd*. Do not compute the numerical values but rather use these relationships:

E + E = E   E + O = O   O + E = O   O + O = E

ow

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

CONNECTING FRACTALS to the MIDDLE SCHOOL CURRICULUM

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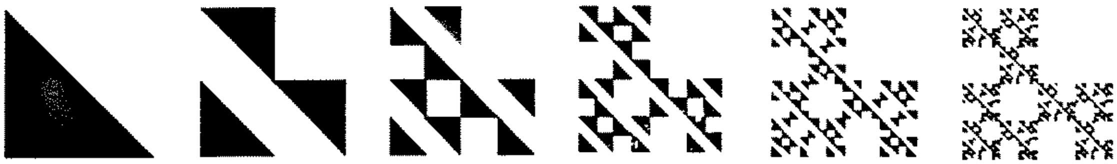
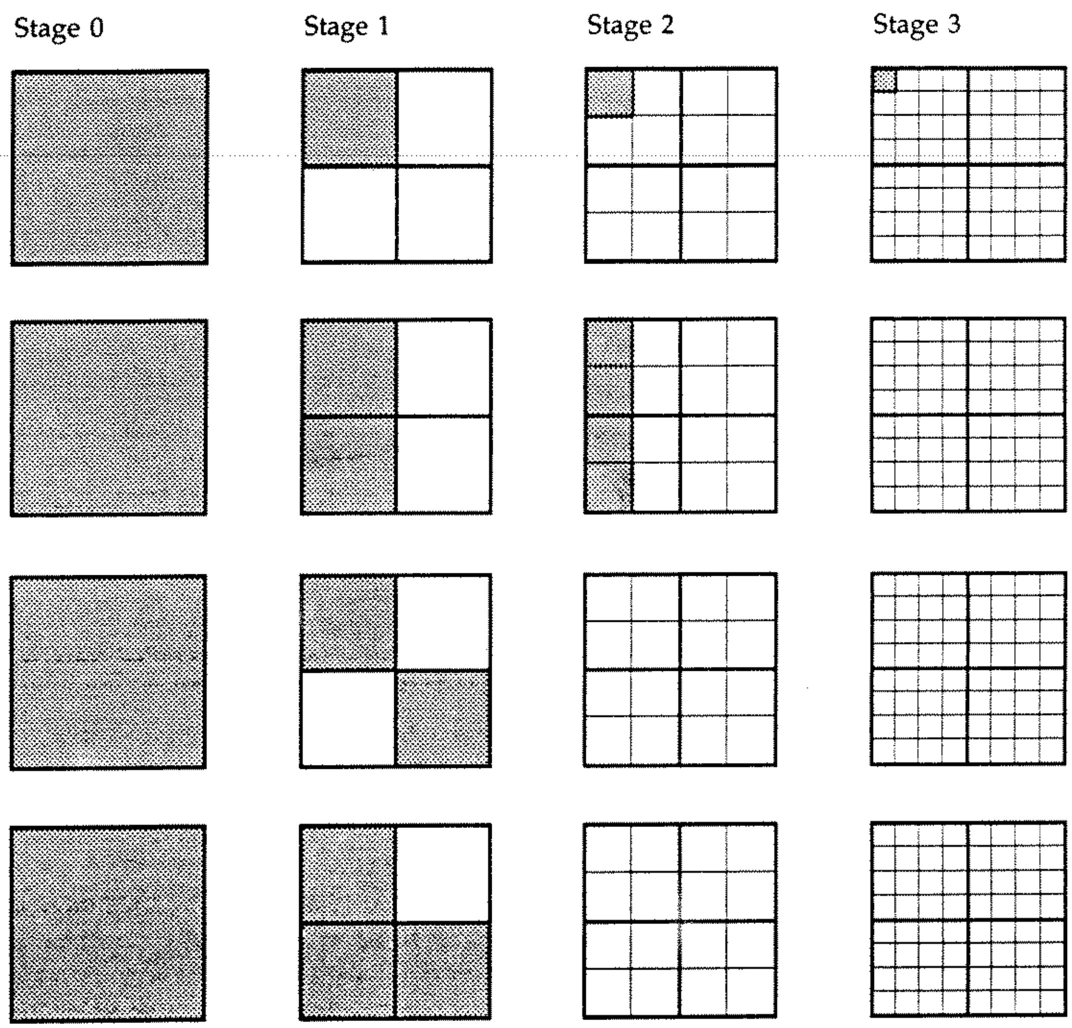
Montclair State University
Upper Montclair, New Jersey
07043



Granite School District
Salt Lake City, Utah

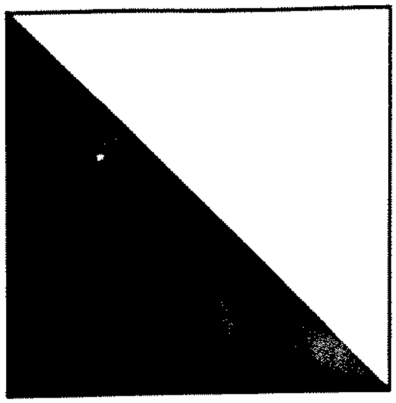
March 1996

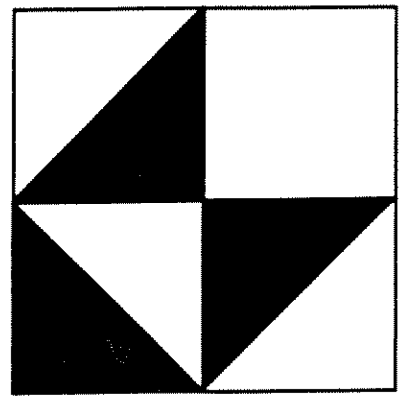


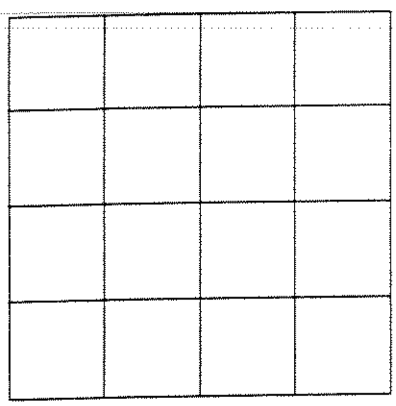
BASE	COPY	SCALE	OFFSET HORI	OFFSET VERT	ORIENT-ATION
2	1	1/2	0	1/2	I
	2	1/2	0	0	I
	3	1/2	1/2	0	I

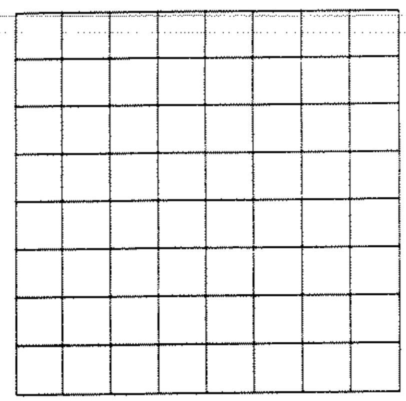
REDUCE, REPLICATE, REBUILD
Rotations in Square Grids

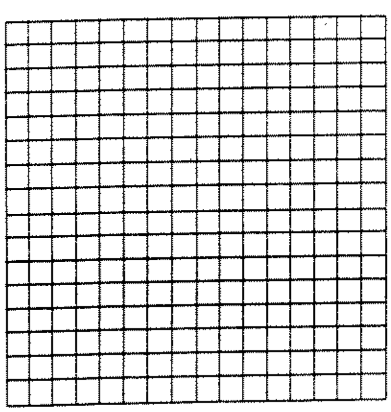
Dr. Evan Maletsky
Montclair State University

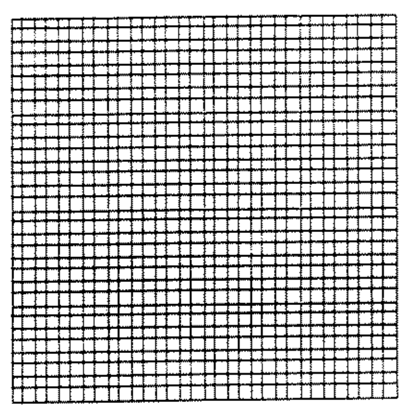




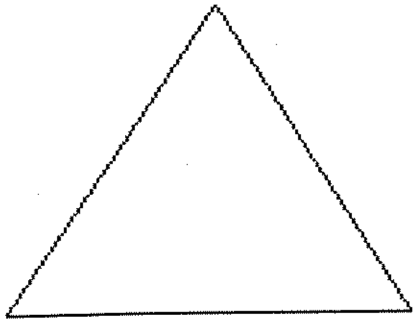








Cynthia Lanius



The Koch Snowflake

You may print and use this triangle grid paper to help you with this drawing.

Step One.

Start with a large equilateral triangle.

Step Two.

Make a Star.

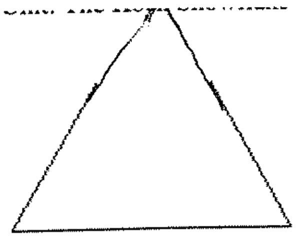
1. Divide one side of the triangle into three equal parts and remove the middle section.
2. Replace it with two lines the same length as the section you removed.
3. Do this to all three sides of the triangle.

Do it again and again.

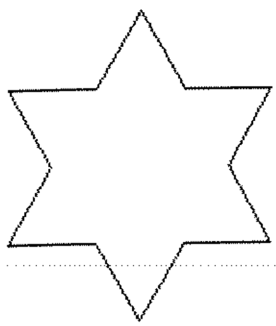
Do it infinitely many times and you have a fractal.

Want to take a long, careful look at what it looks like?

See a few of the steps below.



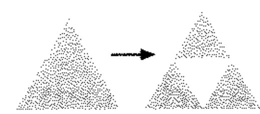
Step One



Step Two

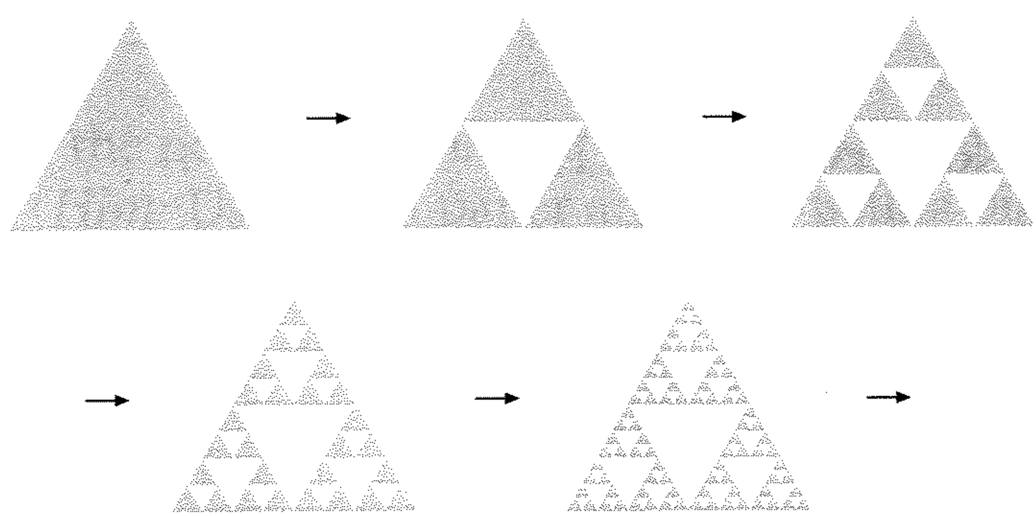
Before describing exactly what a fractal is, let's first look at a number of different examples of fractals. There are several different methods you can use to construct a fractal. In this lesson the methods are **deterministic**. That means that our fractals will be constructed according to specific rules that are set down ahead of time. In another lesson you will investigate random fractals, where the rules are quite different.

THE SIERPIŃSKI TRIANGLE



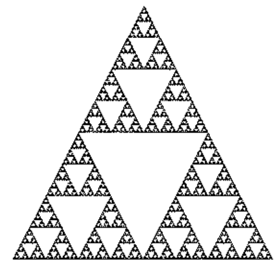
In one of the standard methods of constructing fractals, the iteration rule involves successive "removals." To construct the shape known as the Sierpiński triangle, we begin with an equilateral triangle as the seed and then use the iteration rule: "Remove from the middle of this triangle a smaller triangle with side lengths one-half those of the original so that three congruent triangles remain." Geometrically, this iteration rule is expressed by the images above.

The orbit is then:



There is one technical, but important, detail. When you remove each triangle, you actually remove only the triangle's interior, not its boundary. The sides and vertices of each removed triangle remain as part of the next figure in the orbit.

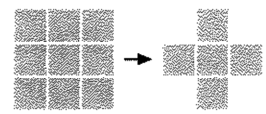
When this rule is iterated infinitely often, you reach a limiting shape known as the Sierpiński triangle. This is one of the simplest examples of the geometric objects known as **fractals.**

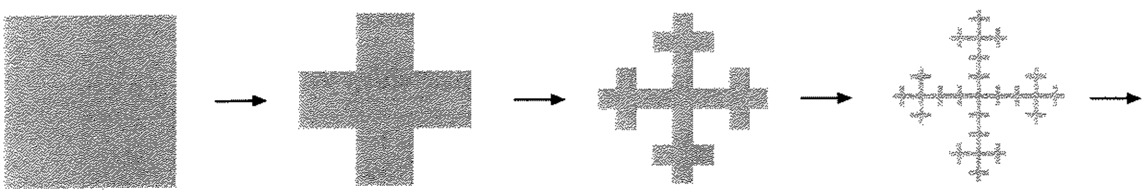


Let's just look at a few more geometric objects constructed in this manner.

THE FRACTAL PLUS

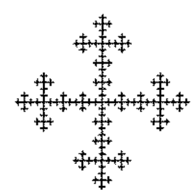
Here is another geometric iteration that results in a fractal. Start with a square whose side has length 1. The iteration rule is to first divide the square into nine squares whose sides have length ⅓. Then remove the four corner squares as shown here:





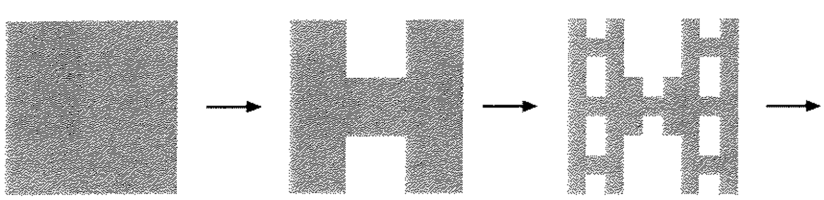
After this first iteration, five squares remain. Each of them has sides of length ⅓ and area ⅑. Now we iterate. At the next iteration we find 25 squares, each with sides of length ⅑ and area 1/81. At the third iteration we have 125 squares with sides of length 1/27, and so forth.

If we continue this process forever, the squares get smaller and the eventual image is a "fractal plus sign" like the one at right.

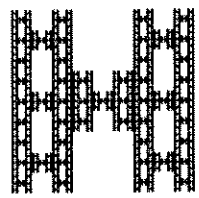


THE FRACTAL H

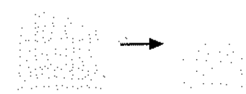
Here is a similar iteration rule. As done for the Fractal Plus, we begin with a square that has been subdivided into nine smaller squares. This time we remove only two of the smaller squares, leaving behind the shape of an "H." Here are the first three figures in this orbit:



When we apply this rule over and over, a new fractal results:



A NON-FRACTAL



Not all processes involving removals lead to fractal images. For example, consider the following iteration rule. Start with a square whose sides have length 1. Then divide the square horizontally into two congruent rectangles. Each will have length 1 and width ½. Remove the upper rectangle, leaving behind the bottom rectangle.

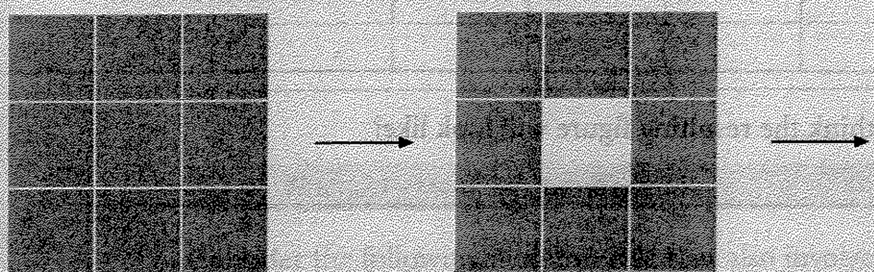
After one more iteration the remaining rectangle has width ¼.

If we continue the process indefinitely, the shape approaches a segment whose length is the same as the side of the original square. Whatever a fractal is, it is not a segment, so *this* process of removals does *not* generate a fractal.

All of the fractals in this section were generated by removing certain pieces of "regular" geometric figures. We will encounter other ways of constructing fractals in other lessons.

1 ▸ THE SIERPIŃSKI CARPET

The Sierpiński carpet is generated by the following iteration rule: Begin with a square whose side has length 1. Then divide the square into nine equal-sized squares whose sides have length ⅓. Remove the middle square, leaving eight smaller squares behind.



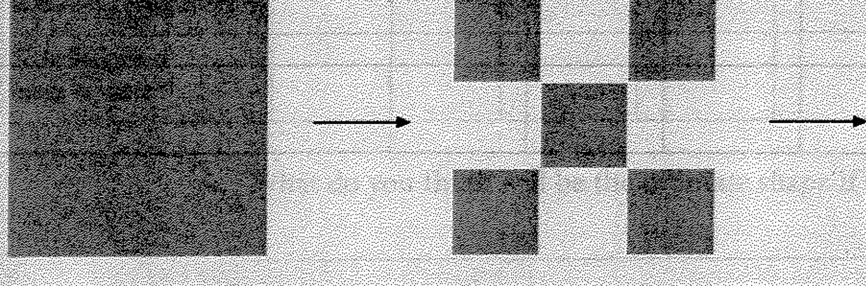
a. Draw the next figure in this orbit.

b. Now complete the table below.

	Original square (the seed)	First iteration	Second iteration	Third iteration	*n*th iteration
Total number of squares remaining	1	8			
Side length of each square	1	$\frac{1}{3}$			

2 ▸ THE FRACTAL X

What happens when we consider the following modification of the preceding iteration rule? Again, start with a square of length 1. Break this square into nine pieces as above, but this time remove four squares as shown here:



a. Draw the next figure in this orbit.

b. Now complete the table below.

	Original square (the seed)	First iteration	Second iteration	Third iteration	*n*th iteration
Total number of squares remaining	1				
Side length of each square	1				

c. What do you think the resulting figure will look like?

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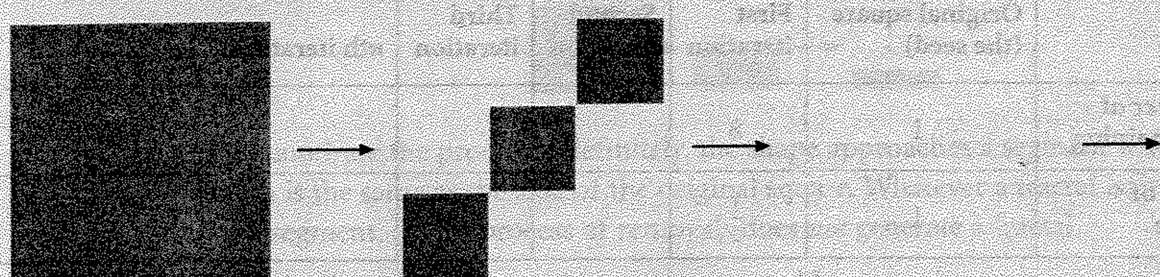
3 ▹ THE DIAGONAL

Suppose we modify the above iteration rule so that only three squares lying on a diagonal line remain after the removals, as in this picture:



a. Draw the next two figures in this orbit.

b. Now complete the table below.

	Original square (the seed)	First iteration	Second iteration	Third iteration	*n*th iteration
Total number of squares remaining	1				
Side length of each square	1				