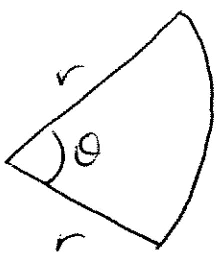
3.4 (continued)

Ex 4 A flower bed will be in the shape of a sector of a circle (a pre-shaped region) of radius r & vertex angle θ. Find r & θ if its area is a constant A & the perimeter is a minimum.

goal: $r, \theta = ?$

minimize perimeter



$$P = r + r + \left(\frac{\theta}{2\pi}\right) 2\pi r$$

$$P = 2r + \theta r$$

☆ $\frac{\theta}{2\pi}$ is fraction of circle (or percent)

$$A = \frac{\theta}{2\pi}(\pi r^2)$$

$$A = \frac{\theta r^2}{2}$$

A is a constant so, solve for θ

$$\theta = \frac{2A}{r^2}$$

$$P = 2r + \left(\frac{2A}{r^2}\right) r$$

$$P = 2r + \frac{2A}{r}$$

$$P' = 2 - \frac{2A}{r^2} \qquad r \neq 0$$

$$2 - \frac{2A}{r^2} = 0$$

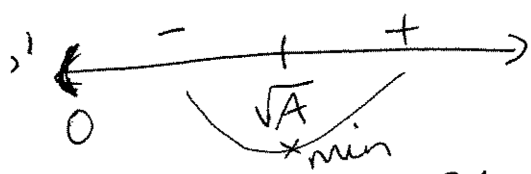
$$2 = \frac{2A}{r^2}$$

$$2r^2 = 2A$$

$$r^2 = A$$

$$r = \pm\sqrt{A}$$

(but r must be positive, so throw away $r = -\sqrt{A}$)



test $r = \frac{\sqrt{A}}{2}$ $\quad 2 - \frac{2A}{\frac{A}{4}} = 2 - 8 < 0$

$r = 2\sqrt{A}$ $\quad 2 - \frac{2A}{4A} > 0$

$\Rightarrow$ $r = \sqrt{A}$ and

$$\theta = \frac{2A}{(\sqrt{A})^2} = \frac{2A}{A} = 2$$

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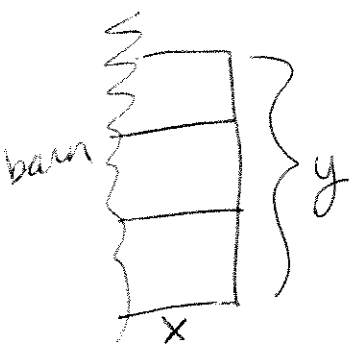
Ex 6 A farmer has 80 ft of fence. He needs to enclose three identical pens along one side of his barn (the side along the barn needs no fence). What dimensions for the total enclosure make the area of the pens as large as possible?

maximize area



Perimeter of fence → (given)

$$P = 4x + y = 80 \text{ ft}$$

$$y = 80 - 4x$$

$$A = xy = x(80-4x)$$

$$A = 80x - 4x^2$$

$$A = -4x^2 + 80x$$

this is concave down parabola so we'll get max pt when derivative is zero

$$A' = -8x + 80 = 0$$

$$8x = 80$$

$$x = 10 \text{ ft}$$

$$\Rightarrow \quad y = 80 - 4(10) = 40$$

10 ft by 40 ft