

## 5.1 Using Fundamental Identities

(★ table pg 374)

Ex 1 Given  $\csc \theta = -\frac{5}{3}$  and  $\tan \theta > 0$ ,  
find all six trig fn values.

Ex 2 Simplify  $\frac{\cot^2 \theta}{\csc^2 \theta}$

5.1 (cont)

Ex 3 Simplify  $(1 - \cos^2 x) \csc x$ .

Ex 4 Simplify  $\cos^2 x + \cos^2 x \tan^2 x$

5.1 (cont)

Ex 5 Simplify.

(a)  $\frac{\cos^2 x - 4}{\cos x + 2}$

(b)  $\tan x - \frac{\sec^2 x}{\tan x}$

(c)  $\frac{\tan^2 x}{\csc x + 1}$

## 5.1 (cont)

Ex 6 Use the trig substitution to write the algebraic expression as a trig expression.

$$\sqrt{x^2 - 4}$$

$$\text{let } x = 2 \sec \theta$$

$$(0 < \theta < \pi/2)$$

## 5.2 Verifying Trig Identities

- Vocab
- ① expression
  - ② equation
  - ③ identity

Steps for verifying identity  $\Rightarrow$

(1) start w/ one side

(2) legally manipulate it to look like other side

(\* we usually choose "uglier" side to start w/)

Ex 1 Verify the identity  $\cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1$ .

## 5.2 (cont)

Ex 2 Verify the identities.

$$(a) \sec^6 x (\sec x + \tan x) - \sec^4 x (\sec x + \tan x) = \sec^5 x \tan^3 x$$

$$(b) \sec^2 \left( \frac{\pi}{2} - x \right) - 1 = \cot^2 x$$

5,2 (cont)

Ex 3 Verify the identities.

$$(a) \frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$$

$$(b) 1 - \sin \theta \cos \theta = \frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta}$$

## 5-3 Solving Trig Equations

Ex 1

Solve

$$\tan^2 x = 1.$$

Ex 2

Solve for  $x$ .

$$\sin^2 x = 3\cos^2 x$$



### 5.3 (cont)

Ex 3 Solve for  $x$ .

(a)  $\sec x \csc x = 2 \csc x$

(b)  $\sec x + \tan x = 1$

S.3 (cont)

Ex 4 Solve for  $x$ .

(a)  $\sin^2(3x) - 2\sin(3x) + 1 = 0$  assume  $x \in [0, 2\pi)$

(b)  $\cos x + \sin x \tan x = 2$

## 5.4 Sum And Difference Formulas

$$\sin(u+v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Ex 1 Find exact value of  $\cos 15^\circ$ .

Ex 2 Verify the identity  $\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$ .

5.4 (cont)

Ex 3 Write as an algebraic expression

$$\sin(\arctan 2x - \arccos x)$$

Ex 4 Simplify  $\tan(\pi + \theta)$

5.4 (cont)

Ex 5 Solve for  $x \in [0, 2\pi)$   
 $\tan(x+\pi) + 2\sin(x+\pi) = 0$

5.4 (cont)

Ex 6 Find the exact value of  $\csc(u-v)$  given that  $\sin u = \frac{5}{13}$  and  $\cos v = \frac{-3}{5}$  (in Q2).

## 5.5 Multiple-Angle and Product-to-Sum Formulas

### Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta}$$

### Power Reducing Formulas

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

### Half Angle Formulas

$$\sin(\theta/2) = \pm \sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos(\theta/2) = \pm \sqrt{\frac{1 + \cos\theta}{2}}$$

$$\begin{aligned}\tan(\theta/2) &= \frac{1 - \cos\theta}{\sin\theta} \\ &= \frac{\sin\theta}{1 + \cos\theta}\end{aligned}$$

### Product to Sum Formulas

$$\sin u \sin v = \frac{1}{2} [\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u+v) - \sin(u-v)]$$

### Sum to Product Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\begin{aligned}\cos x - \cos y &= \\ &= -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\end{aligned}$$

## 5.5 (cont)

Ex 1 Rewrite as a sum  $2 \sin x \sin y$ .

Ex 2 Solve for  $x \in [0, 2\pi)$   $4 \sin x \cos x = 1$



5.5 (cont)

Ex 3 Find exact value of  $\sin(2u)$  and  $\cos(2u)$  if  $\cot u = -5$ , and  $u \in (\frac{3\pi}{2}, 2\pi)$ .

Ex 4 Rewrite as a sum or difference  
 $6 \sin 45^\circ \cos 15^\circ$

## S.5 (cont)

Ex 5 Write as a product  
 $\sin(x + \frac{\pi}{2}) + \sin(x - \frac{\pi}{2})$ .

Ex 6 Rewrite  $\sin^4 x \cos^4 x$  in terms of 1<sup>st</sup>  
power of  $\cos x$ .

S.S (cont)

Ex 7 Verify the identity

$$\frac{\cos 3x}{\cos x} = 1 - 4\sin^2 x$$

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