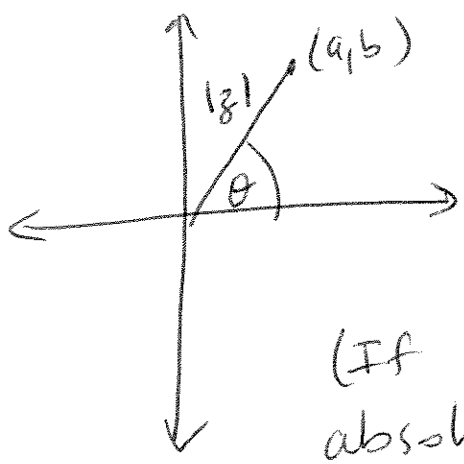
6.5 Trigonometric Form of a Complex

To graph a complex #, we do it on a 2d graph; horizontal axis = $\mathbb{R}$ axis
vertical axis = imaginary axis



$\Rightarrow$ pt $(a,b) = a+bi$

If $z = a+bi \in \mathbb{C}$,

then $|z| = |a+bi| = \sqrt{a^2+b^2}$

(If $b=0$, then $|z| = |a|$, i.e. the absolute value of a $\mathbb{R}$ #.)

Also, $(a,b) = (r\cos\theta, r\sin\theta)$

$$\Rightarrow \boxed{\begin{aligned} a+bi &= r\cos\theta + i(r\sin\theta) = z, \\ |z| &= r = \sqrt{a^2+b^2} \\ \tan\theta &= \frac{b}{a} \end{aligned}}$$

r is called modulus of z, θ is argument of z

<u>Ex 1</u> Plot $z = -3+4i$ and find $|z|$.

6.5 (cont)

Ex 2 Write $5(\cos 135° + i \sin 135°)$ in standard form (i.e. $a+bi$).

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

Then $z_1 z_2 = [r_1(\cos\theta_1 + i\sin\theta_1)][r_2(\cos\theta_2 + i\sin\theta_2)]$

$= r_1 r_2[\cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\cos\theta_2\sin\theta_1 - \sin\theta_1\sin\theta_2]$

$= r_1 r_2[(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\sin\theta_2\cos\theta_1 + \sin\theta_1\cos\theta_2)]$

$$\boxed{z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]}$$

Product Rule (for complex #s)

6.5 (cont)

Also $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$ if $z_2 \neq 0$

$\Rightarrow$ By Product Rule, if $z = r(\cos\theta + i\sin\theta)$

$$z^2 = zz = r^2(\cos(2\theta) + i\sin(2\theta))$$
$$z^3 = z^2 z = r^3(\cos(3\theta) + i\sin(3\theta))$$
$$z^4 = z^3 z = r^4(\cos(4\theta) + i\sin(4\theta))$$
$$\vdots$$
$$\boxed{z^n = r^n(\cos(n\theta) + i\sin(n\theta))} \quad n \in \mathbb{N}$$

De Moivre's Thm

Ex 3 Simplify.

(a) $[2(\cos 5^\circ + i\sin 5^\circ)][3(\cos 20^\circ + i\sin 20^\circ)]$

6.5 (cont)

Ex 3 (cont)

(b) $\dfrac{6(\cos 40^\circ + i\sin 40^\circ)}{7(\cos 100^\circ + i\sin 100^\circ)}$

Ex 4 Find trig form of $4 - 4\sqrt{3}\, i$.

$\boxed{\text{HW: } 6.5 \# 1\text{-}39 \text{ odd, } 45\text{-}87 \text{ odd}}$