

Math1060 Midterm 2

Fall, 2007

Instructor: Kelly MacArthur

Name: _____

Key

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work given. Don't hesitate to ask a question if you don't understand the instructions for any question.

(1) (15 pts) Solve this equation. $\sin^2 x + \frac{1}{2} \cos x = 1$ (Give **all** solutions.)

$$\begin{aligned} \sin^2 x = 1 - \cos^2 x &\Rightarrow 1 - \cos^2 x + \frac{1}{2} \cos x = 1 \\ -\cos^2 x + \frac{1}{2} \cos x &= 0 \\ \cos x (-\cos x + \frac{1}{2}) &= 0 \end{aligned}$$

$$\begin{aligned} \cos x = 0 &\quad \text{or} \quad -\cos x + \frac{1}{2} = 0 \\ x = \frac{\pi}{2}, \frac{3\pi}{2} &\quad \cos x = \frac{1}{2} \end{aligned}$$

$$x = \frac{\pi}{2} + 2n\pi$$

$$x = \frac{3\pi}{2} + 2n\pi$$

$$x = \frac{\pi}{3}, -\frac{\pi}{3}$$

$$\frac{\pi}{3} + 2n\pi$$

$$-\frac{\pi}{3} + 2n\pi$$

$$x = \frac{\pm \pi}{3} + 2n\pi \quad \text{or} \quad \frac{\pi}{2} + 2n\pi \quad \text{or} \quad \frac{3\pi}{2} + 2n\pi$$

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(2) (12 pts) Simplify. $(\tan x + 1)^2 \cos x = (\tan x + 1)(\tan x + 1) \cos x$

$$= (\tan^2 x + 2\tan x + 1) \cos x$$

$$= (\sec^2 x + 2\tan x) \cos x$$

$$= \left(\frac{1}{\cos^2 x} + \frac{2\sin x}{\cos x} \right) \cos x$$

$$= \frac{1}{\cos x} + 2\sin x$$

$$= \sec x + 2\sin x$$

but

$$\tan^2 x + 1 = \sec^2 x$$

Answer 2: $\sec x + 2\sin x$

(3) (12 pts) Verify this identity. $2\sec^2 x - \cos^2 x - 2\sec^2 x \sin^2 x - \sin^2 x = 1$

$$2\sec^2 x - \cos^2 x - 2\sec^2 x \sin^2 x - \sin^2 x$$

$$= (2\sec^2 x - 2\sec^2 x \sin^2 x) - (\cos^2 x + \sin^2 x)$$

$$= 2\sec^2 x (1 - \sin^2 x) - 1$$

$$= 2\sec^2 x (\cos^2 x) - 1$$

$$= 2\left(\frac{1}{\cos^2 x}\right)(\cos^2 x) - 1 = 2 - 1 = 1$$

(4) (12 pts) Verify this identity.

$$\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y}$$

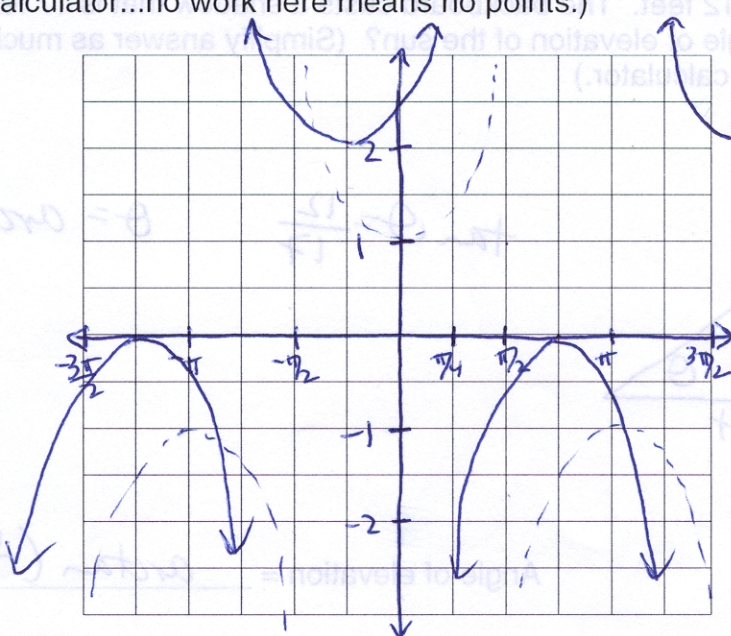
$$= \frac{(\cos x - \cos y)(\cos x + \cos y) + (\sin x - \sin y)(\sin x + \sin y)}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$= \frac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)} = \frac{(\cos^2 x + \sin^2 x) - (\cos^2 y + \sin^2 y)}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$= \frac{1 - 1}{(\sin x + \sin y)(\cos x + \cos y)}$$

$$= 0$$

(5) (12 pts) Sketch the graph of $y = \sec\left(x - \frac{\pi}{4}\right) + 1$. (Show all your work without a calculator...no work here means no points.)



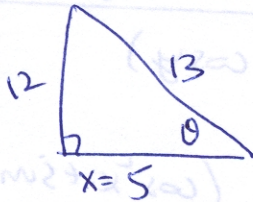
is $y = \sec$
 we want
 shift
 and
 left $\pi/4$

(6) Find the exact values of the expressions, **without** a calculator.

(a) (12 pts) $\cot(\arcsin(\frac{-12}{13})) = \underline{\underline{-5/12}}$

$\theta = \arcsin(\frac{-12}{13})$ in

$\theta = \frac{-12}{13}$



Quadrant $\Rightarrow \cot \theta < 0$

$\Rightarrow \cot \theta = \frac{-5}{12}$

$x^2 + 12^2 = 13^2$

$x^2 + 144 = 169$

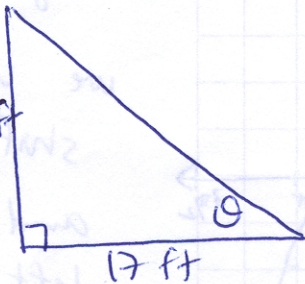
$x^2 = 25 \Rightarrow x = 5$

(b) (10 pts) $\tan^{-1}(-1) = \underline{\underline{-\pi/4}}$

$\tan \theta = -1$ when $\theta = 3\pi/4$ or $-\pi/4$

but $\arctan x$ only returns val between $-\pi/2$ and $\pi/2 \Rightarrow \tan^{-1}(-1)$

(6) (15 pts) The height of an outdoor basketball backboard at my brother's house is 12 feet. The backboard casts a shadow that is 17 feet long. What is the angle of elevation of the sun? (Simplify answer as much as possible without a calculator.)



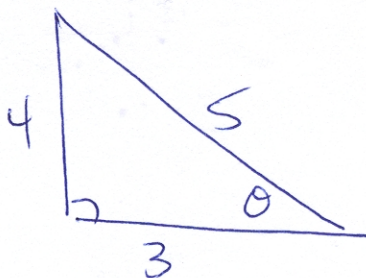
$\tan \theta = \frac{12}{17}$

$\theta = \arctan \frac{12}{17}$

Angle of elevation = $\underline{\underline{\arctan(\frac{12}{17})}}$

Extra Credit: (5 pts) Find the remaining five trigonometric function values of θ if

$\cos \theta = \frac{3}{5}$ and $\tan \theta < 0 \Rightarrow$ in Quadrant 4



$\sin \theta = \underline{\underline{-4/5}}$

$\csc \theta = \underline{\underline{-5/4}}$

$\cos \theta = \frac{3}{5}$

$\sec \theta = \underline{\underline{5/3}}$

$\tan \theta = \underline{\underline{-4/3}}$

$\cot \theta = \underline{\underline{-3/4}}$