

Math1060 Final Exam
 Fall, 2007
 Instructor: Kelly MacArthur

Name: Key

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work given. Simplify all answers.

- (1) (20 pts) For the vectors $\vec{u} = \langle 8, -2 \rangle$ and $\vec{v} = 4\hat{i} + 5\hat{j}$, answer the following questions.

(a) Find $3\vec{v} - \vec{u}$.

$$3\langle 8, -2 \rangle - \langle 4, 5 \rangle = \langle 24, -6 \rangle - \langle 4, 5 \rangle = \langle 20, -11 \rangle$$

$$3\vec{v} - \vec{u} = \underline{\langle 20, -11 \rangle \text{ or } 20\hat{i} - 11\hat{j}}$$

(b) Find $\|\vec{u}\|$.

$$\|\vec{u}\| = \sqrt{8^2 + (-2)^2} = \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17}$$

$$\|\vec{u}\| = \underline{2\sqrt{17}}$$

(c) Find \hat{u} .

$$\hat{u} = \frac{\vec{u}}{\|\vec{u}\|} = \frac{\langle 8, -2 \rangle}{2\sqrt{17}} = \left\langle \frac{8}{2\sqrt{17}}, \frac{-2}{2\sqrt{17}} \right\rangle = \left\langle \frac{4}{\sqrt{17}}, \frac{-1}{\sqrt{17}} \right\rangle$$

$$\hat{u} = \underline{\left\langle \frac{4}{\sqrt{17}}, \frac{-1}{\sqrt{17}} \right\rangle}$$

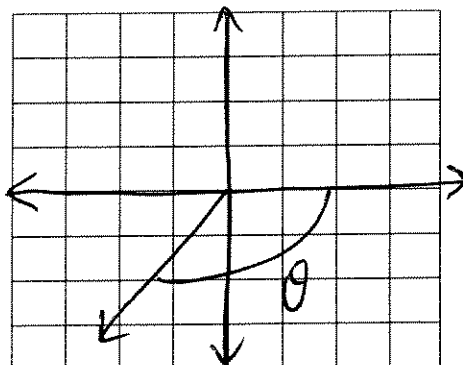
(d) Find $\vec{u} \cdot \vec{v}$.

$$\vec{u} \cdot \vec{v} = 8(4) + (-2)(5) = 32 - 10 = 22$$

$$\vec{u} \cdot \vec{v} = \underline{22}$$

2. (25 pts) For the angle $\theta = -120^\circ$, answer the following questions.

(a) Sketch the angle in standard position.



(b) Determine a positive coterminal angle for θ .

$$-120^\circ + 360^\circ = 240^\circ$$

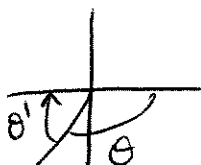
$$\text{coterminal angle} = \underline{240^\circ}$$

(c) Convert this angle, θ , to radians.

$$\theta = -120^\circ \left(\frac{\pi}{180^\circ} \right) = \frac{-2\pi}{3}$$

$$\theta \text{ in radians} = \underline{-\frac{2\pi}{3}}$$

(d) Find the reference angle, θ' (in degrees).



$$\theta' = 180^\circ - 120^\circ = 60^\circ$$

$$\theta' \text{ (in degrees)} = \underline{60^\circ}$$

(e) Find the exact values (no calculators) of the six trigonometric functions of θ .

$$\sin \theta = \underline{-\frac{\sqrt{3}}{2}}$$

$$\cos \theta = \underline{-\frac{1}{2}}$$

$$\sec \theta = \underline{-2}$$

$$\csc \theta = \underline{-\frac{2}{\sqrt{3}}}$$

$$\tan \theta = \underline{\sqrt{3}}$$

$$\cot \theta = \underline{\frac{1}{\sqrt{3}}}$$

3. (20 pts) Verify these identities.

(a) $\frac{\cos(3x)}{\cos x} = 1 - 4\sin^2 x$

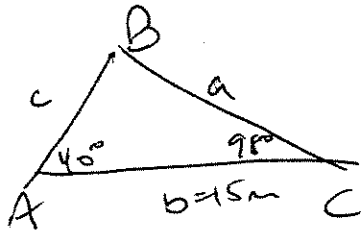
$$\begin{aligned}\frac{\cos(3x)}{\cos x} &= \frac{\cos(x+2x)}{\cos x} = \frac{\cos x \cos 2x - \sin x \sin(2x)}{\cos x} \\ &= \frac{\cos x (1 - 2\sin^2 x) - \sin x (2\sin x \cos x)}{\cos x} \\ &= \frac{\cancel{\cos x} [1 - 2\sin^2 x - 2\sin^2 x]}{\cancel{\cos x}} \\ &= 1 - 4\sin^2 x\end{aligned}$$

(b) $\frac{1}{\sec x + 1} - \frac{1}{\sec x - 1} = -2\cot^2 x$

$$\begin{aligned}\frac{1}{\sec x + 1} - \frac{1}{\sec x - 1} &= \left(\frac{1}{\sec x + 1}\right)\left(\frac{\sec x - 1}{\sec x - 1}\right) - \left(\frac{1}{\sec x - 1}\right)\left(\frac{\sec x + 1}{\sec x + 1}\right) \\ &= \frac{\cancel{\sec x} - 1 - \cancel{\sec x} - 1}{(\sec x + 1)(\sec x - 1)} = \frac{-2}{\sec^2 x - 1} \\ &= \frac{-2}{\tan^2 x} = -2\cot^2 x\end{aligned}$$

4. (10 pts) Determine the area of the triangle ABC with given information.

$A=40^\circ$, $C=98^\circ$, $b=15$ meters



by law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

we know

$$B = 180^\circ - 40^\circ - 98^\circ$$

$$B = 42^\circ$$

$$\Rightarrow \frac{a}{\sin 40^\circ} = \frac{15}{\sin 42^\circ} \Rightarrow a = \frac{15 \sin 40^\circ}{\sin 42^\circ}$$

$$a \approx 14.4$$

$$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} (14.4)(15) \sin 98^\circ \approx 107$$

Area = 107 m²

5. (15 pts) For the function $f(x) = -3 \sin\left(\frac{\pi}{2}x\right) + 4$, answer the following questions.

(a) Compared to the base graph of $y = \sin x$ there is Horizontal and/or Vertical shifting:

shifted **Right** or **Left** or **neither** (circle one)
by units

shifted **Up** or **Down** or **neither** (circle one)
by 4 units

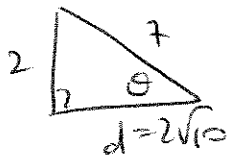
(b) Amplitude = 3

(c) Period = 4 $2\pi \left(\frac{2}{\pi}\right) = 4$

6. (30 pts) Find the exact value of these expressions (without a calculator!).

(a) $\csc\left(\arcsin\left(\frac{2}{7}\right)\right) = \underline{\underline{7/2}}$

Let $\theta = \arcsin\left(\frac{2}{7}\right)$



$$\begin{aligned} 2^2 + d^2 &= 7^2 \\ d^2 &= 40 \\ d &= 2\sqrt{10} \end{aligned}$$

$$\csc \theta = \frac{7}{2}$$

(b) $\cot\left(\frac{5\pi}{3}\right) = \underline{\underline{-1/\sqrt{3}}}$

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \quad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\begin{aligned} \cot\left(\frac{5\pi}{3}\right) &= \frac{1/2}{-\sqrt{3}/2} \\ &= -\frac{1}{\sqrt{3}} \end{aligned}$$

(c) $\arccos\left(\frac{-\sqrt{3}}{2}\right) = \underline{\underline{5\pi/6}}$

(arccos fn only returns values between 0 and π)

(d) $\arctan(\tan(-135^\circ)) = \underline{\underline{45^\circ \text{ or } \pi/4}}$

$$\tan(-135^\circ) = 1 \quad \arctan 1 = 45^\circ$$

(e) $\sin\left(\arcsin\left(\frac{3}{4}\right)\right) = \underline{\underline{3/4}}$

7. (10 pts) Solve the equation. Give all solutions in the interval $\beta \in [0, 2\pi)$.

$$\cot^2 \beta = \cot \beta$$

$$\cot^2 \beta - \cot \beta = 0$$

$$\cot \beta (\cot \beta - 1) = 0$$

$$\cot \beta = 0$$

$$\Rightarrow \cos \beta = 0$$

$$\beta = \pi/2, 3\pi/2$$

$$\cot \beta - 1 = 0$$

$$\cot \beta = 1$$

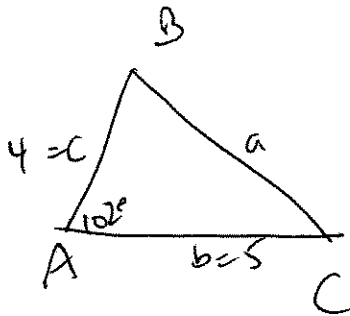
$$\beta = \pi/4, 5\pi/4$$

$$\cot \beta = \frac{\cos \beta}{\sin \beta}$$

$$\beta = \underline{\pi/2, 3\pi/2, \pi/4, 5\pi/4}$$

8. (10 pts) Use the given information to solve the triangle (give angles in degrees and round to nearest whole numbers).

$$A = 102^\circ, \quad c = 4, \quad b = 5$$



Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$= 5^2 + 4^2 - 2(5)(4) \cos 102^\circ$$

$$a^2 \approx 49.3 \Rightarrow a \approx 7.02$$

$$\frac{\sin C}{4} = \frac{\sin 102^\circ}{7.02}$$

$$\sin C = \frac{4 \sin 102^\circ}{7.02}$$

$$\Rightarrow C \approx 33.9^\circ$$

$$B = \underline{44^\circ}$$

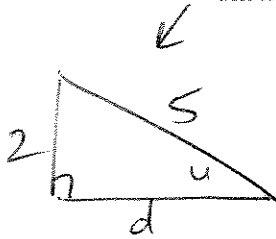
$$C = \underline{34^\circ}$$

$$a = \underline{7}$$

$$\Rightarrow B = 180^\circ - 102^\circ - 33.9^\circ = 44.1^\circ$$

9. (30 pts) Find the exact value of the following trigonometric functions given that

$$\sin u = \frac{-2}{5}, \quad \cos v = \frac{-1}{3}, \quad \text{and both } u \text{ and } v \text{ are in Quadrant 3.}$$



$$d^2 + 4 = 25 \Rightarrow d = \sqrt{21}$$

$$(a) \sin(u-v) = \frac{2 - 2\sqrt{42}}{15}$$



$$1^2 + x^2 = 9$$

$$x^2 = 8$$

$$x = 2\sqrt{2}$$

$$\Rightarrow \cos u = \frac{-\sqrt{21}}{5}$$

$$\sin v = \frac{-2\sqrt{2}}{3}$$

$$\tan v = \frac{-2\sqrt{2}}{3}$$

$$\frac{-1}{3} = 2\sqrt{2}$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$= \frac{-2}{5} \left(\frac{-1}{3} \right) - \left(\frac{-\sqrt{21}}{5} \right) \left(\frac{-2\sqrt{2}}{3} \right) = \frac{2 - 2\sqrt{42}}{15}$$

$$(b) \tan(2v) = \frac{-4\sqrt{2}}{7}$$

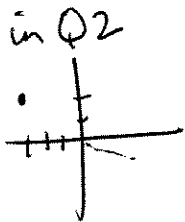
$$\tan(2v) = \frac{2 \tan v}{1 - \tan^2 v} = \frac{2(2\sqrt{2})}{1 - (2\sqrt{2})^2} = \frac{4\sqrt{2}}{1 - 4(2)} = \frac{4\sqrt{2}}{-7}$$

$$(c) \cos^2\left(\frac{u}{2}\right) = \frac{5 - \sqrt{21}}{10}$$

$$\cos^2\left(\frac{u}{2}\right) = \frac{1 + \cos u}{2} = \frac{\left(1 + \frac{-\sqrt{21}}{5}\right) \left(\frac{5}{5}\right)}{2} = \frac{5 - \sqrt{21}}{10}$$

10. (20 pts) For $z = -3 + 2i$, answer the following questions.

(a) Find the trigonometric form of z , i.e. in form $z = r(\cos\theta + i\sin\theta)$. (Give θ in degrees and round to the nearest degree. Give r in exact form.)



$$r = \sqrt{(-3)^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

$$\tan \theta = \frac{2}{-3} \quad \arctan\left(\frac{-2}{3}\right) \approx -33.7^\circ$$

$$\Rightarrow \theta = 180^\circ - (33.7^\circ) \approx 146.3^\circ$$

$$z = \underline{\underline{\sqrt{13} (\cos(146^\circ) + i \sin(146^\circ))}}$$

(b) Find z^4 and write answer in standard form, $a+bi$.

$$z^4 = (-3+2i)^4 = \sqrt{13}^4 (\cos(4(146^\circ)) + i \sin(4(146^\circ)))$$

$$= 169 (\cos 584^\circ + i \sin 584^\circ)$$

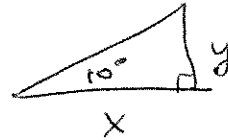
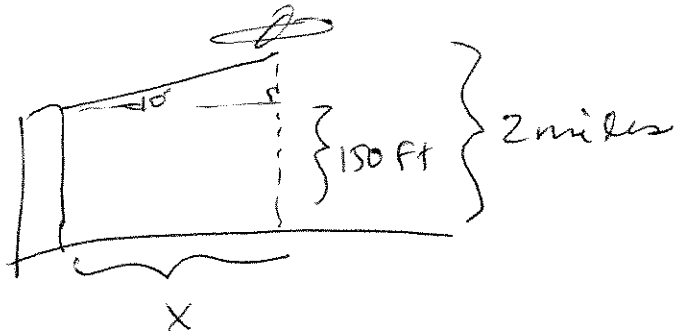
$$\approx 169 (-0.7193 + i(-0.6947))$$

$$\approx -121.56 - 117.4i$$

$$z^4 = \underline{\underline{-121.56 - 117.4i}}$$

11. (10 pts) The angle of elevation from the top of an observation tower to a plane is 10 degrees. The plane is two miles high and the height of the observation tower is 150 feet. What is the horizontal distance from the observation tower to the plane?

$x = ?$



$$y = 2 \text{ miles} - 150 \text{ ft}$$

$$y = 2 \text{ mi} \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) - 150 \text{ ft}$$

$$y = 10560 - 150$$

$$y = 10410 \text{ ft}$$

$$\tan 10^\circ = \frac{10410}{x}$$

$$x = \frac{10410}{\tan 10^\circ} \approx 59038 \text{ ft}$$

$$59038 \text{ ft} \left(\frac{1 \text{ mi}}{5280 \text{ ft}} \right) = 11.2 \text{ mi} \quad \text{Horizontal distance} = \frac{59038 \text{ ft}}{11.2 \text{ miles}} \quad (\text{or } 11.2 \text{ miles})$$

Extra Credit: (10 pts) Find the exact value of $\cos(15^\circ)$ (no calculator).

$$\cos^2(15^\circ) = \cos^2\left(\frac{30^\circ}{2}\right) = \frac{1 + \cos 30^\circ}{2}$$

$$= \left(\frac{1 + \frac{\sqrt{3}}{2}}{2} \right) \left(\frac{2}{2} \right)$$

$$= \frac{2 + \sqrt{3}}{4}$$

$$\Rightarrow \cos 15^\circ = \sqrt{\frac{2 + \sqrt{3}}{4}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$\cos(15^\circ) = \frac{\sqrt{2 + \sqrt{3}}}{2}$$