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Math1060 Final Exam

Fall, 2007
Instructor: Kelly MacArthur

Name: Key

Instructions: Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work given. Simplify all answers.

(1) *(20 pts)* For the vectors $\vec{u}=\langle 6,-3\rangle$ and $\vec{v}=5\hat{i}+4\hat{j}$, answer the following questions.

(a) Find $3\vec{v}-\vec{u}$.

$3\langle 5,4\rangle-\langle 6,-3\rangle=\langle 15-6,12-(-3)\rangle=\langle 9,15\rangle$

$3\vec{v}-\vec{u}=$ $\langle 9,15\rangle$ or $9\hat{i}+15\hat{j}$

(b) Find $\|\vec{u}\|$.

$\|\vec{u}\|=\sqrt{6^2+(-3)^2}=\sqrt{36+9}=\sqrt{45}=3\sqrt{5}$

$\|\vec{u}\|=$ $3\sqrt{5}$

(c) Find $\hat{u}$.

$\hat{u}=\dfrac{\vec{u}}{\|\vec{u}\|}=\dfrac{\langle 6,-3\rangle}{3\sqrt{5}}=\left\langle \dfrac{6}{3\sqrt{5}},\dfrac{-3}{3\sqrt{5}}\right\rangle=\left\langle \dfrac{2}{\sqrt{5}},\dfrac{-1}{\sqrt{5}}\right\rangle$

$\hat{u}=$ $\langle 2/\sqrt{5},-1/\sqrt{5}\rangle$

(d) Find $\vec{u}\cdot\vec{v}$.

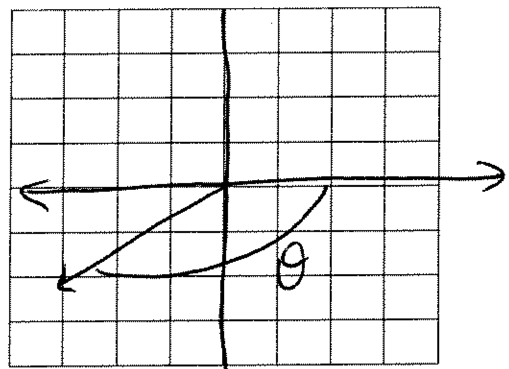
$\vec{u}\cdot\vec{v}=\langle 6,-3\rangle\cdot\langle 5,4\rangle$

$=6(5)+(-3)(4)=30-12=18$

$\vec{u}\cdot\vec{v}=$ 18

2. *(25 pts)* For the angle $\theta = -150^\circ$, answer the following questions.

(a) Sketch the angle in standard position.



(b) Determine a positive coterminal angle for θ .

$-150^\circ + 360^\circ = 210^\circ$

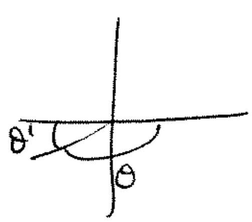
coterminal angle = 210°

(c) Convert this angle, θ , to radians.

$-150^\circ \left(\frac{\pi}{180^\circ} \right) = \frac{-5\pi}{6}$

θ in radians = $-5\pi/6$

(d) Find the reference angle, θ' (in degrees).



$\theta' = 30^\circ$

θ' (in degrees)= 30°

(e) Find the exact values (no calculators) of the six trigonometric functions of θ .

$\sin\theta$ = $-1/2$ $\cos\theta$ = $-\sqrt{3}/2$

$sec\,\theta$ = $-2/\sqrt{3}$ $csc\,\theta$ = -2

$\tan\theta$ = $1/\sqrt{3}$ $\cot\theta$ = $\sqrt{3}$

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3. *(20 pts)* Verify these identities.

(a) $\frac{1}{\sec x+1}-\frac{1}{\sec x-1}=-2\cot^2 x$

$$\frac{1}{\sec x+1}-\frac{1}{\sec x-1}=\left(\frac{1}{\sec x+1}\right)\left(\frac{\sec x-1}{\sec x-1}\right)-\left(\frac{1}{\sec x-1}\right)\left(\frac{\sec x+1}{\sec x+1}\right)$$

$$=\frac{\sec x-1-(\sec x+1)}{(\sec x-1)(\sec x+1)}=\frac{\cancel{\sec x}-1-\cancel{\sec x}-1}{\sec^2 x-1}$$

$\tan^2 x+1=\sec^2 x$

$$=\frac{-2}{\tan^2 x}=-2\cot^2 x$$

(b) $\frac{\cos(3x)}{\cos x}=1-4\sin^2 x$

$$\frac{\cos(3x)}{\cos x}=\frac{\cos(x+2x)}{\cos x}=\frac{\cos x\cos 2x-\sin x\sin(2x)}{\cos x}$$

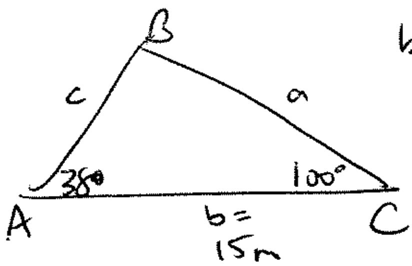
$$=\frac{\cos x(1-2\sin^2 x)-\sin x(2\sin x\cos x)}{\cos x}$$

$$=\frac{\cancel{\cos x}[1-2\sin^2 x-2\sin^2 x]}{\cancel{\cos x}}$$

$$=1-4\sin^2 x$$

4. *(10 pts)* Determine the area of the triangle ABC with given information.

$A = 38°$, $C = 100°$, $b = 15$ meters



by Law of Sines

we know

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$B = 180° - 100° - 38°$

$\Rightarrow B = 42°$

$$\Rightarrow \frac{a}{\sin 38°} = \frac{15}{\sin 42°} \Rightarrow a = \frac{15 \sin 38°}{\sin 42°}$$

$a \approx 13.8$

$$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2}(13.8)(15)\sin 100°$$

$$\simeq 102$$

Area = $102\ m^2$

5. *(15 pts)* For the function $f(x) = -5\sin\left(\frac{\pi}{4}x\right) + 2$, answer the following questions.

(a) Compared to the base graph of $y = \sin x$ there is Horizontal and/or Vertical shifting:

shifted **Right** or **Left** or **neither** (circle one)

by ______ units

shifted **Up** or **Down** or **neither** (circle one)

by 2 units

(b) Amplitude = 5

(c) Period = 8

$$2\pi\left(\frac{4}{\pi}\right) = 8$$

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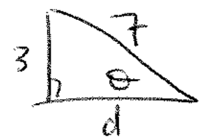
6. *(30 pts)* Find the **exact** value of these expressions (without a calculator!).

(a) $csc(\arcsin\left(\frac{3}{7}\right)) =$ $7/3$

let $\theta = \arcsin\left(\frac{3}{7}\right) \Leftrightarrow \sin\theta = \frac{3}{7}$



$\Rightarrow \csc\theta = \frac{1}{\sin\theta} = \frac{1}{3/7} = 7/3$

$3^2 + d^2 = 7^2$

$d^2 = 40$

$d = 2\sqrt{10}$

(b) $\tan\left(\frac{5\pi}{3}\right) =$ $-\sqrt{3}$

$\sin(5\pi/3) = -\sqrt{3}/2$ $\cos(5\pi/3) = \frac{1}{2}$ $\tan(5\pi/3) = \frac{-\sqrt{3}/2}{1/2}$

$= \sqrt{3}$

(c) $\arcsin\left(\frac{-\sqrt{3}}{2}\right) =$ $-\pi/3$

(remember arcsin fn only returns values between $-\pi/2$ and $\pi/2$)

(d) $\arctan(\tan(135°)) =$ $-45°$ or $-\pi/4$

$\tan 135° = -1$ $\arctan(-1) = -45°$ or $-\pi/4$

(e) $\cos(\arccos\left(\frac{3}{4}\right)) =$ $3/4$

7. *(10 pts)* Solve the equation. Give all solutions in the interval $\beta \in [0, 2\pi)$.

$$\cot\beta = \cot^2\beta$$

$\cot\beta = \frac{\cos\beta}{\sin\beta}$

$\cot^2\beta - \cot\beta = 0$

$\cot\beta(\cot\beta - 1) = 0$

$\cot\beta = 0$ or $\cot\beta - 1 = 0$

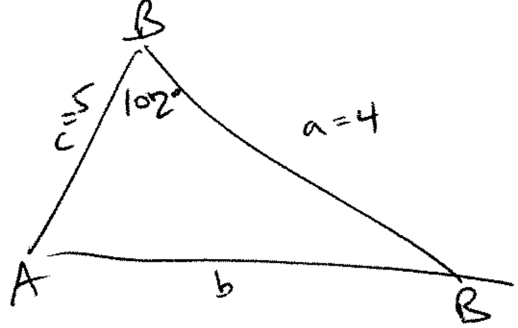
$\Rightarrow \cos\beta = 0$ $\quad\quad$ $\cot\beta = 1$

$\beta = \pi/2, 3\pi/2$ $\quad\quad$ $\beta = \pi/4, 5\pi/4$

$\beta =$ $\pi/4, 5\pi/4, \pi/2, 3\pi/2$

8. *(10 pts)* Use the given information to solve the triangle (give angles in degrees and round to nearest whole numbers).

$$B = 102^\circ, \quad a = 4, \quad c = 5$$



Law of Cosines

$b^2 = 4^2 + 5^2 - 2(4)(5)\cos 102^\circ$

$b^2 \approx 49.3 \Rightarrow b \approx 7.02$

$\frac{\sin A}{4} = \frac{\sin 102^\circ}{7.02}$

$\sin A = \frac{4 \sin 102^\circ}{7.02}$

$A = 33.9^\circ$ $\quad \Rightarrow C = 180^\circ - 33.9^\circ - 102^\circ = 44.1^\circ$

A = 34°

C = 44°

b = 7

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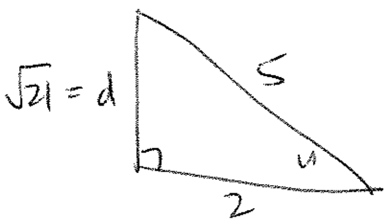
9. *(30 pts)* Find the exact value of the following trigonometric functions given that

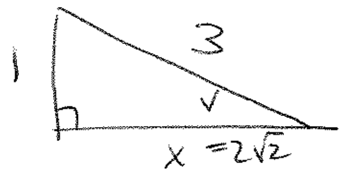
$\cos u = \frac{-2}{5}$, $\sin v = \frac{-1}{3}$, and both u and v are in Quadrant 3.



$d^2 + 4 = 25$

$d = \sqrt{21}$

$\Rightarrow \sin u = \frac{-\sqrt{21}}{5}$



$1 + x^2 = 9$

$x^2 = 8$

$x = 2\sqrt{2}$

$\Rightarrow \cos v = \frac{-2\sqrt{2}}{3}$

$\tan v = \frac{1}{2\sqrt{2}}$

(a) $\cos(u-v)$ = $\boxed{\frac{4\sqrt{2} + \sqrt{21}}{5}}$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$= \left(\frac{-2}{5}\right)\left(\frac{-2\sqrt{2}}{3}\right) + \left(\frac{-\sqrt{21}}{5}\right)\left(\frac{-1}{3}\right)$$

$$= \frac{4\sqrt{2}}{15} + \frac{\sqrt{21}}{15}$$

(b) $\tan(2v)$ = $\boxed{\frac{4\sqrt{2}}{7} \text{ or } \frac{8}{7\sqrt{2}}}$

$$\tan(2v) = \frac{2\tan v}{1 - \tan^2 v} = \frac{2\left(\frac{1}{2\sqrt{2}}\right)}{1 - \left(\frac{1}{2\sqrt{2}}\right)^2} = \frac{\frac{1}{\sqrt{2}}}{1 - \frac{1}{8}}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{7}{8}} = \frac{1}{\sqrt{2}} \div \frac{7}{8}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{8}{7}$$

$$= \frac{8}{7\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{4\sqrt{2}}{7}$$

(c) $\cos^2\left(\frac{u}{2}\right)$ = $\boxed{\frac{3}{10}}$

$$\cos^2\left(\frac{u}{2}\right) = \frac{1 + \cos u}{2}$$

$$= \frac{1 + \frac{-2}{5}}{2} = \frac{3/5}{2} = \frac{3}{10}$$

10. *(20 pts)* For $z=-2+3i$, answer the following questions.

(a) Find the trigonometric form of z , i.e. in form $z=r(\cos\theta+i\sin\theta)$. (Give θ in degrees and round to the nearest degree. Give r in <u>exact</u> form.)

in Q2

$$r=\sqrt{(-2)^2+3^2} = \sqrt{4+9} = \sqrt{13}$$

$$\tan\theta = \frac{3}{-2}$$

we need θ in Quad 2

$$\arctan\left(\frac{-3}{2}\right) \approx -56^\circ \quad \Rightarrow \quad \theta = 180^\circ - 56^\circ = 124^\circ$$

z = $\sqrt{13}\left(\cos(124^\circ) + i\sin(124^\circ)\right)$

(b) Find z^4 and write answer in standard form, $a+bi$.

$$z^4 = r^4(\cos 4\theta + i\sin 4\theta)$$

$$= (\sqrt{13})^4\left(\cos(4(124^\circ)) + i\sin(4(124^\circ))\right)$$

$$\approx 169(-0.7193 + 0.6947i)$$

$$= -121.6 + 117.4i$$

z^4 = $-121.6 + 117.4i$

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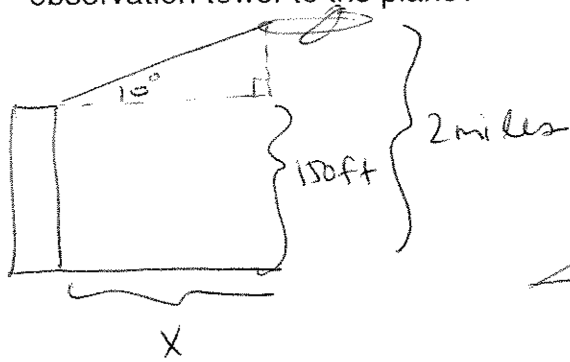
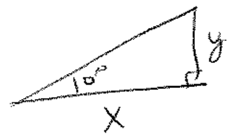
11. *(10 pts)* The angle of elevation from the top of an observation tower to a plane is 10 degrees. The plane is two miles high and the height of the observation tower is 150 feet. What is the horizontal distance from the observation tower to the plane?

$X = ?$





$y = 2\text{ miles} - 150\text{ ft}$

$y = 2\text{ mi}\left(\frac{5280\text{ ft}}{1\text{ mi}}\right) - 150\text{ ft}$

$y = 10560 - 150$

$y = 10410\text{ ft}$

$\tan 10° = \frac{10410}{X}$

$X = \frac{10410}{\tan 10°} \approx 59038\text{ ft}$

$59038\text{ft}\left(\frac{1\text{ mi}}{5280\text{ ft}}\right) \approx 11.2\text{ miles}$

Horizontal distance = 59038 ft or 11.2 miles

Extra Credit: *(10 pts)* Find the exact value of $\sin(15°)$ (no calculator).

$\sin^2(15°) = \sin^2\left(\frac{30°}{2}\right) = \frac{1-\cos 30°}{2} = \frac{1-\sqrt{3}/2}{2}\left(\frac{2}{2}\right)$

$\sin^2(15°) = \frac{2-\sqrt{3}}{4}$

$\sin 15° = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

$\sin(15°) = \frac{\sqrt{2-\sqrt{3}}}{2}$