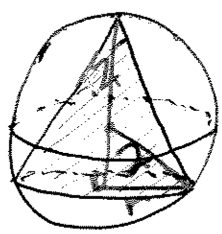
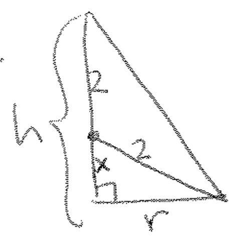
3.4 (cont)

Ex 4 (#30) Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 2.



We have 

this triangle.

$0 \le r \le 2$

$\Rightarrow x^2 + r^2 = 2^2$ (Pythagorean Thm)

$x^2 = 4 - r^2$

$x = \sqrt{4-r^2}$ (since x must be positive)

and $h = x + 2 = \sqrt{4-r^2} + 2$

We want to maximize volume of cone.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \left(\sqrt{4-r^2} + 2\right)$

$\Leftrightarrow V = \frac{1}{3}\pi r^2\left(\sqrt{4-r^2}\right) + \frac{2}{3}\pi r^2$

$V' = \frac{2}{3}\pi r\left(\sqrt{4-r^2}\right) + \frac{1}{3}\pi r^2\left(\frac{1}{\cancel{2}}(4-r^2)^{-1/2}(-\cancel{2}r)\right) + \frac{4}{3}\pi r$

$= \frac{2}{3}\pi r\left(\sqrt{4-r^2}\right) - \frac{\pi r^3}{3\sqrt{4-r^2}} + \frac{4}{3}\pi r = 0$

$\frac{3}{\pi}\sqrt{4-r^2}\left(\frac{2\pi r}{3}\sqrt{4-r^2} - \frac{\pi r^3}{3\sqrt{4-r^2}} + \frac{4\pi}{3}r\right) = 0\left(\frac{3}{\pi}\sqrt{4-r^2}\right)$

$2r(4-r^2) - r^3 + 4r\sqrt{4-r^2} = 0$

$8r - 2r^3 - r^3 + 4r\sqrt{4-r^2} = 0$

$\frac{4r\sqrt{4-r^2}}{4r} = \frac{3r^3 - 8r}{4r}$

Math 100
67

(turn over)

$$\sqrt{4-r^2} = \frac{3}{4}r^2 - 2$$

$$\left(\sqrt{4-r^2}\right)^2 = \left(\frac{3}{4}r^2 - 2\right)^2$$

$$4-r^2 = \frac{9}{16}r^4 - 3r^2 + 4$$

$$-4 + r^2 \qquad\qquad +r^2 \quad -4$$

$$16\,(0) = \left(\frac{9}{16}r^4 - 2r^2\right)16$$

$$0 = 9r^4 - 32r^2$$

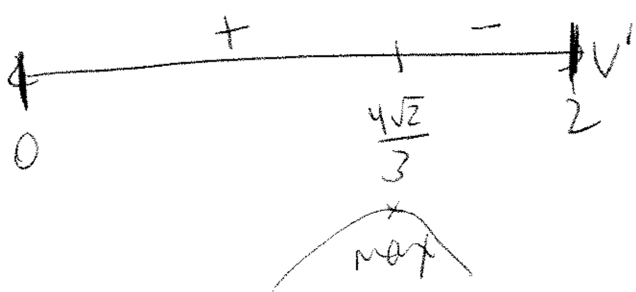
$$0 = r^2(9r^2 - 32)$$

$$r = 0 \quad \text{or} \quad 9r^2 - 32 = 0$$

$$9r^2 = 32$$

$$r^2 = 32/9$$

$$r = \sqrt{32/9} = \frac{4\sqrt{2}}{3} \simeq 1.89$$



$$V'(r) = \frac{\pi}{3}\left(2r\sqrt{4-r^2} - \frac{r^3}{\sqrt{4-r^2}} + 4r\right)$$

$$V'(1) = \frac{\pi}{3}\left(2\sqrt{3} - \frac{1}{\sqrt{3}} + 4\right) > 0$$

$$V'(1.999) \simeq -118 < 0$$

max at

$$\Rightarrow r = \frac{4\sqrt{2}}{3} \Rightarrow h = 2 + \sqrt{4 - (32/9)}$$

$$= 2 + \sqrt{4/9} = 2 + 2/3 = 8/3$$

$$\Rightarrow V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{32}{9}\right)\left(\frac{8}{3}\right) = \frac{256}{81}\pi$$

max volume

math1100

67A

Ex2 (#8) Find # of units x that produces the minimum avg cost per unit $\bar{C}$.

$$C(x) = 0.02x^3 + 55x + 1250$$

$$\Rightarrow \bar{C}(x) = \frac{C(x)}{x} = 0.02x^2 + 55 + \frac{1250}{x}$$

$$\bar{C}'(x) = 0.04x - \frac{1250}{x^2} = 0$$

$$x^2\,(0.04x) = \left(\frac{1250}{x^2}\right)x^2$$

$$\frac{0.04x^3}{0.04} = \frac{1250}{0.04}$$

$$x^3 = 31250$$

$$\sqrt[3]{x^3} = \sqrt[3]{31250}$$

$$x = 25\sqrt[3]{2} \approx 31.5$$

$-$ $+$ $\bar{C}'(x)$

0 $\quad 25\sqrt[3]{2}$

min

$\bar{C}'(1) = 0.04 - 1250 < 0$

$\bar{C}'(100) = 4 - 0.1250 > 0$

min avg cost at $x = 25\sqrt[3]{2}$ but since x is the # of units, it should be a whole #, so check avg cost at $x = 31$ and at $x = 32$.

$\bar{C}(31) = 114.5425806$

$\bar{C}(32) = 114.5425$

$\Rightarrow \boxed{\text{min at } x = 32}$