meaningless form 0/0. In such cases, algebraic manipulation such as factoring and cancellation may give a form from which the limit can be determined.

If $f(x)$ approaches L as x approaches a from the right, then we write $\lim_{x\to a^+} f(x) = L$. If $f(x)$ approaches L as x approaches a from the left, we write $\lim_{x\to a^-} f(x) = L$. These limits are called one-sided limits.

The infinity symbol ∞, which does not represent a number, is used in describing limits. The statement

$$\lim_{x\to\infty} f(x) = L$$

means that as x increases without bound, the values of $f(x)$ approach the number L. A similar statement applies for the situation when $x \to -\infty$, which means that x is decreasing without bound. In general, if $p > 0$, then

$$\lim_{x\to\infty} \frac{1}{x^p} = 0 \quad \text{and} \quad \lim_{x\to-\infty} \frac{1}{x^p} = 0$$

If $f(x)$ increases without bound as $x \to a$, then we write $\lim_{x\to a} f(x) = \infty$. Similarly, if $f(x)$ decreases without bound, we have $\lim_{x\to a} f(x) = -\infty$. To say that the limit of a function is ∞ (or $-\infty$) does not mean that the limit exists. Rather, it is a way of saying that the limit does not exist and tells *why* there is no limit.

There is a rule for evaluating the limit of a rational function (quotient of polynomials) as $x \to \infty$ or $-\infty$. If $f(x)$ is a rational function and $a_n x^n$ and $b_m x^m$ are the terms in the numerator and denominator, respectively, with the greatest powers of x, then

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{a_n x^n}{b_m x^m} \quad \text{and} \quad \lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{a_n x^n}{b_m x^m}$$

In particular, as $x \to \infty$ or $-\infty$, the limit of a polynomial is the same as the limit of the term that involves the greatest power of x. This means that, for a nonconstant polynomial, the limit as $x \to \infty$ or $-\infty$ is either ∞ or $-\infty$.

When interest is compounded at every instant of time, we say that it is compounded continuously. Under continuous compounding at an annual rate r for t years, the formula $S = Pe^{rt}$ gives the compound amount S of a principal of P dollars. The formula $P = Se^{-rt}$ gives the present value P of S dollars. The effective rate corresponding to an annual rate r compounded continuously is $e^r - 1$.

A function f is continuous at a if and only if

1. $f(x)$ is defined at $x = a$
2. $\lim_{x\to a} f(x)$ exists
3. $\lim_{x\to a} f(x) = f(a)$

Geometrically this means that the graph of f has no break when $x = a$. If a function is not continuous at a and is defined on an open interval containing a, except perhaps at a itself, then the function is said to be discontinuous at a. Polynomial functions are continuous everywhere, and rational functions are discontinuous only at points where the denominator is zero.

To solve the inequality $f(x) > 0$ [or $f(x) < 0$], we first find the real zeros of f and the values of x for which f is discontinuous. These values determine intervals, and on each interval, $f(x)$ is either always positive or always negative. To find the sign on any one of these intervals, it suffices to find the sign of $f(x)$ at any point there. After the signs are determined for all intervals, it is easy to give the solution of $f(x) > 0$ [or $f(x) < 0$].

Review Problems

Problem numbers shown in color indicate problems suggested for use as a practice chapter test.

In Problems 1–28, find the limits if they exist. If the limit does not exist, so state that, or use the symbol ∞ or $-\infty$ where appropriate.

1. $\lim_{x\to-1} (2x^2 + 6x - 1)$

2. $\lim_{x\to0} \frac{2x^2 - 3x + 1}{2x^2 - 2}$

3. $\lim_{x\to3} \frac{x^2 - 9}{x^2 - 3x}$

4. $\lim_{x\to-3} \frac{x + 1}{x^2 - 3}$

5. $\lim_{h\to0} (x + h)$

6. $\lim_{x\to2} \frac{x^2 - 4}{x^2 - 3x + 2}$

7. $\lim_{x\to-4} \frac{x^3 + 4x^2}{x^2 + 2x - 8}$

8. $\lim_{x\to1} \frac{x^2 + x - 2}{x^2 + 4x - 5}$

9. $\lim_{x\to\infty} \frac{2}{x + 1}$

10. $\lim_{x\to\infty} \frac{x^2 + 1}{2x^2}$

11. $\lim_{x\to\infty} \frac{5x + 2}{9x - 3}$

12. $\lim_{x\to-\infty} \frac{1}{x^4}$

13. $\lim_{t\to3} \frac{2t - 3}{t - 3}$

14. $\lim_{x\to-\infty} \frac{x^6}{x^5}$

15. $\lim_{x\to-\infty} \frac{x + 3}{1 - x}$

16. $\lim_{x\to9} \sqrt{16}$

17. $\lim_{x\to\infty} \frac{x^2 - 1}{(3x + 2)^2}$

18. $\lim_{x\to1} \frac{x^2 + x - 2}{x - 1}$

19. $\lim_{x\to3^-} \frac{x + 3}{x^2 - 9}$

20. $\lim_{x\to2} \frac{2 - x}{x - 2}$

21. $\lim_{x\to\infty} \sqrt{3x}$

22. $\lim_{y\to5^+} \sqrt{y - 5}$

23. $\lim_{x\to\infty} \frac{x^{100} + (1/x^2)}{e - x^{98}}$

24. $\lim_{x\to-\infty} \frac{ex^2 - x^4}{31x - 2x^3}$

25. $\lim_{x\to1} f(x)$ if $f(x) = \begin{cases} x^2 & \text{if } 0 \le x < 1 \\ x & \text{if } x > 1 \end{cases}$

26. $\lim_{x\to3} f(x)$ if $f(x) = \begin{cases} x + 5 & \text{if } x < 3 \\ 6 & \text{if } x \ge 3 \end{cases}$

27. $\lim_{x\to 4^+} \frac{\sqrt{x^2-16}}{4-x}$ (*Hint:* For $x > 4$, $\sqrt{x^2-16} = \sqrt{x-4}\sqrt{x+4}$)

28. $\lim_{x\to 3^+} \frac{x^2-x-6}{\sqrt{x-3}}$ (*Hint:* For $x > 3$, $\frac{x-3}{\sqrt{x-3}} = \sqrt{x-3}$]

29. If $f(x) = 8x-2$, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

30. If $f(x) = 2x^2-3$, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

31. **Host-Parasite Relationship** For a particular host-parasite relationship, it was determined that when the host density (number of hosts per unit of area) is x, then the number of hosts parasitized over a certain period of time is

$$y = 23\left(1 - \frac{1}{1+2x}\right)$$

If the host density were to increase without bound, what value would y approach?

32. **Predator-Prey Relationship** For a particular predator-prey relationship, it was determined that the number y of prey consumed by an individual predator over a period of time was a function of the prey density x (the number of prey per unit of area). Suppose

$$y = f(x) = \frac{10x}{1+0.1x}$$

If the prey density were to increase without bound, what value would y approach?

33. For an annual interest of 5% compounded continuously, find
    (a) the compound amount of \$2500 after 14 years.
    (b) the present value of \$2500 due in 14 years.

34. For an annual interest rate of 6% compounded continuously, find
    (a) the compound amount of \$800 after 11 years.
    (b) the present value of \$800 due in 11 years.

35. Find the effective rate equivalent to an annual rate of 6% compounded continuously.

36. Find the effective rate equivalent to an annual rate of 1% compounded continuously.

37. If interest is earned at the annual rate of 6.5% compounded continuously, determine the exact number of years required for a principal of P to double.

38. **Investment** The Smiths have just made two investments: \$400 in account A, which earns interest at the rate of 6% compounded semiannually, and \$400 in account B, which earns interest at the rate of 5.5% compounded continuously.
    (a) Calculate the effective interest rate for each investment.
    (b) Which investment is worth more at the end of 5 years and by how much?

39. Using the definition of continuity, show that the function $f(x) = x+5$ is continuous at $x = 7$.

40. Using the definition of continuity, show that the function $f(x) = \frac{x-5}{x^2+2}$ is continuous at $x = 5$.

41. State whether $f(x) = \frac{x}{4}$ is continuous everywhere. Give a reason for your answer.

42. State whether $f(x) = x^2-2$ is continuous everywhere. Give a reason for your answer.

In Problems 43–50, find the points of discontinuity (if any) for each function.

43. $f(x) = \frac{x^2}{x+3}$

44. $f(x) = \frac{0}{x^3}$

45. $f(x) = \frac{x-1}{2x^2+3}$

46. $f(x) = (5-7x)^5$

47. $f(x) = \frac{4-x^2}{x^2+3x-4}$

48. $f(x) = \frac{2x+6}{x^3+x}$

49. $f(x) = \begin{cases} x+4 & \text{if } x > -2 \\ 3x+6 & \text{if } x \le -2 \end{cases}$

50. $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 1 \\ 1 & \text{if } x \ge 1 \end{cases}$

In Problems 51–58, solve the given inequalities.

51. $x^2+4x-12 > 0$

52. $3x^2-3x-6 \le 0$

53. $x^3 \ge 2x^2$

54. $x^3+8x^2+15x \ge 0$

55. $\frac{x+5}{x^2-1} < 0$

56. $\frac{x(x+5)(x+8)}{3} < 0$

57. $\frac{x^2+3x}{x^2+2x-8} \ge 0$

58. $\frac{x^2-4}{x^2+2x+1} \le 0$

59. Graph $f(x) = \frac{x^3+3x^2-19x+18}{x^3-2x^2+x-2}$. Use the graph to estimate $\lim_{x\to 2} f(x)$.

60. Graph $f(x) = \frac{\sqrt{x+3}-2}{x-1}$. From the graph, estimate $\lim_{x\to 1} f(x)$.

61. Graph $f(x) = x\ln x$. From the graph, estimate the one-sided limit $\lim_{x\to 0^+} f(x)$.

62. Graph $f(x) = \frac{e^x-1}{(e^x+1)(e^{2x}-e^x)}$. Use the graph to estimate $\lim_{x\to 0} f(x)$.

63. Graph $f(x) = x^3-x^2+x-6$. Use the graph to determine the solution of

$$x^3-x^2+x-6 \ge 0$$

64. Graph $f(x) = \frac{x^5-4}{x^3+1}$. Use the graph to determine the solution of

$$\frac{x^5-4}{x^3+1} \le 0$$

In economics, the term *marginal* is used to describe derivatives of specific types of functions. If $c = f(q)$ is a total-cost function (c is the total cost of q units of a product), then the rate of change

$$\frac{dc}{dq} \text{ is called marginal cost}$$

We interpret marginal cost as the approximate cost of one additional unit of output. (Average cost per unit, $\bar{c}$, is related to total cost c by $\bar{c} = c/q$, or $c = \bar{c}q$.)

A total-revenue function $r = f(q)$ gives a manufacturer's revenue r for selling q units of product. (Revenue r and price p are related by $r = pq$.) The rate of change

$$\frac{dr}{dq} \text{ is called marginal revenue}$$

which is interpreted as the approximate revenue obtained from selling one additional unit of output.

If r is the revenue that a manufacturer receives when the total output of m employees is sold, then the derivative dr/dm is called the marginal-revenue product and gives the approximate change in revenue that results when the manufacturer hires an extra employee.

If $C = f(I)$ is a consumption function, where I is national income and C is national consumption, then

$$\frac{dC}{dI} \text{ is marginal propensity to consume}$$

and

$$1 - \frac{dC}{dI} \text{ is marginal prospensity to save}$$

For any function, the relative rate of change of $f(x)$ is

$$\frac{f'(x)}{f(x)}$$

which compares the rate of change of $f(x)$ with $f(x)$ itself. The percentage rate of change is

$$\frac{f'(x)}{f(x)} \cdot 100$$

Review Problems

Problem numbers shown in color indicate problems suggested for use as a practice chapter test.

In Problems 1–4, use the definition of the derivative to find $f'(x)$.

1. $f(x) = 2 - x^2$

2. $f(x) = 2x^2 - 3x + 1$

3. $f(x) = \sqrt{3x}$

4. $f(x) = \dfrac{2}{1+4x}$

In Problems 5–38, differentiate.

5. $y = 7^4$

6. $y = x$

7. $y = 7x^4 - 6x^3 + 5x^2 + 1$

8. $y = 4(x^2+5) - 7x$

9. $f(s) = s^2(s^2+2)$

10. $y = \sqrt{x+3}$

11. $y = \dfrac{x^2+1}{5}$

12. $y = -\dfrac{2}{2x^2}$

13. $y = (x^2+6x)(x^3-6x^2+4)$

14. $y = (x^2+1)^{100}(x-6)$

15. $f(x) = (2x^2+4x)^{100}$

16. $f(w) = w\sqrt{w} + w^2$

17. $y = \dfrac{3}{2x+1}$

18. $y = \dfrac{4x^3 - 8x^2}{5x}$

19. $y = (8+2x)(x^2+1)^4$

20. $g(z) = (2z)^{3/5} + 5$

21. $f(z) = \dfrac{z^2-1}{z^2+4}$

22. $y = \dfrac{x-5}{(x+2)^2}$

23. $y = \sqrt[3]{4x-1}$

24. $f(x) = (1+2^3)^{12}$

25. $y = \dfrac{1}{\sqrt{1-x}}$

26. $y = \dfrac{x(x+1)}{2x^2+3}$

27. $h(x) = (x-6)^4(x+5)^3$

28. $y = \dfrac{(x+3)^5}{x}$

29. $y = \dfrac{5x-4}{x+6}$

30. $f(x) = 8x^2\sqrt{1+2x^3}$

31. $y = 2x^{-3/8} + (2x)^{-3/8}$

32. $y = \sqrt{\dfrac{x}{2}} + \sqrt{\dfrac{2}{x}}$

33. $y = \dfrac{x^2+6}{\sqrt{x^2+5}}$

34. $y = \sqrt[3]{(7-3x^2)^2}$

35. $y = (x^3+6x^2+9)^{3/5}$

36. $z = 0.4x^2(x+1)^{-3} + 0.5$

37. $g(z) = \dfrac{-7z}{(z-1)^{-1}}$

38. $g(z) = \dfrac{-3}{4(z^5+2z-5)^4}$

In Problems 39–42, find an equation of the tangent line to the curve at the point corresponding to the given value of x.

39. $y = x^2 - 6x + 4,\ x = 1$

40. $y = -2x^3 + 6x + 1,\ x = 2$

41. $y = \sqrt[3]{x},\ x = 8$

42. $y = -\dfrac{x}{x-9},\ x = 10$

43. If $f(x) = 4x^2 + 2x + 8$, find the relative and percentage rates of change of $f(x)$ when $x = 1$.

44. If $f(x) = x/(x+4)$, find the relative and percentage rates of change of $f(x)$ when $x = 1$.

45. Marginal Revenue If $r = q(20 - 0.1q)$ is a total-revenue function, find the marginal-revenue function.

46. Marginal Cost If

$$c = 0.0001q^3 - 0.02q^2 + 3q + 6000$$

is a total-cost function, find the marginal cost when $q = 100$.

47. Consumption Function If

$$C = 7 + 0.6I - 0.25\sqrt{I}$$

is a consumption function, find the marginal propensity to consume and the marginal propensity to save when $I = 16$.

48. Demand Equation If $p = \dfrac{q+12}{q+5}$ is a demand equation, find the rate of change of price p with respect to quantity q.

49. Demand Equation If $p = -0.5q + 450$ is a demand equation, find the marginal-revenue function.

50. Average Cost If $\bar{c} = 0.03q + 1.2 + \dfrac{3}{q}$ is an average-cost function, find the marginal cost when $q = 100$.

51. Power-Plant Cost Function The total-cost function of an electric light and power plant is estimated by[18]

$$c = 16.68 + 0.125q + 0.00439q^2 \quad 20 \le q \le 90$$

where q is the eight-hour total output (as a percentage of capacity) and c is the total fuel cost in dollars. Find the marginal-cost function and evaluate it when $q = 70$.

52. Marginal-Revenue Product A manufacturer has determined that m employees will produce a total of q units of product per day, where

$$q = m(50 - m)$$

If the demand function is given by

$$p = -0.01q + 9$$

find the marginal-revenue product when $m = 10$.

53. Winter Moth In a study of the winter moth in Nova Scotia,[19] it was determined that the average number of eggs, y, in a female moth was a function of the female's abdominal width x (in millimeters), where

$$y = f(x) = 14x^3 - 17x^2 - 16x + 34$$

and $1.5 \le x \le 3.5$. At what rate does the number of eggs change with respect to abdominal width when $x = 2$?

54. Host-Parasite Relation For a particular host-parasite relationship, it is found that when the host density (number of hosts per unit of area) is x, the number of hosts that are parasitized is

$$y = 12\left(1 - \frac{1}{1+3x}\right) \quad x \ge 0$$

For what value of x does dy/dx equal $\frac{1}{4}$?

55. Bacteria Growth Bacteria are growing in a culture. The time t (in hours) for the number of bacteria to double in number (the generation time) is a function of the temperature T (in degrees Celsius) of the culture and is given by

$$t = f(T) = \begin{cases} \frac{1}{24}T + \frac{11}{4} & \text{if } 30 \le T \le 36 \\ \frac{4}{3}T - \frac{175}{4} & \text{if } 36 < T \le 39 \end{cases}$$

Find dt/dT when (a) $T = 38$ and (b) $T = 35$.

56. Motion The position function of a particle moving in a straight line is

$$s = \frac{9}{2t^2 + 3}$$

where t is in seconds and s is in meters. Find the velocity of the particle at $t = 1$.

57. Rate of Change The volume of a sphere is given by $V = \frac{1}{6}\pi d^3$, where d is the diameter. Find the rate of change of V with respect to d when $d = 4$ ft.

58. Motion The position function for a ball thrown vertically upward from the ground is

$$s = 218t - 16t^2$$

where s is the height in feet above the ground after t seconds. For what value(s) of t is the velocity 64 ft/s?

59. Find the marginal-cost function if the average-cost function is

$$\bar{c} = 2q + \frac{10{,}000}{q^2}$$

60. Find an equation of the tangent line to the curve

$$y = \frac{(x^3 + 2)\sqrt{x+1}}{x^4 + 2x}$$

at the point on the curve where $x = 1$.

61. A manufacturer has found that when m employees are working, the number of units of product produced per day is

$$q = 10\sqrt{m^2 + 3600} - 600$$

The demand equation for the product is

$$9q + p^2 - 7200 = 0$$

where p is the selling price when the demand for the product is q units per day.

(a) Determine the manufacturer's marginal-revenue product when $m = 80$.

(b) Find the relative rate of change of revenue with respect to the number of employees when $m = 80$.

(c) Suppose it would cost the manufacturer \$300 more per day to hire an additional employee. Would you advise the manufacturer to hire the 81st employee? Why?

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[18] J. A. Nordin, "Note on a Light Plant's Cost Curves," *Econometrica* 15 (1947), 231–55.

[19] D. G. Embree, "The Population Dynamics of the Winter Moth in Nova Scotia, 1954–1962," *Memoirs of the Entomological Society of Canada* no. 46 (1965).

both sides of the equation with respect to x. When doing this, remember that

$$\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$$

Finally, we solve the resulting equation for dy/dx.

The derivative formulas for natural logarithmic and exponential functions are

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

and

$$\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$$

To differentiate logarithmic and exponential functions in bases other than e, you can first transform the function to base e and then differentiate the result. Alternatively, differentiation formulas can be applied:

$$\frac{d}{dx}(\log_b u) = \frac{1}{(\ln b)u} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(a^u) = a^u(\ln a)\frac{du}{dx}$$

Suppose that $f(x)$ consists of products, quotients, or powers. To differentiate $y = \log_b[f(x)]$, it may be helpful to use properties of logarithms to rewrite $\log_b[f(x)]$ in terms of simpler logarithms and then differentiate that form. To differentiate $y = f(x)$, where $f(x)$ consists of products, quotients, or powers, the method of logarithmic differentiation may be used. In that method, we take the natural logarithm of both sides of $y = f(x)$ to obtain $\ln y = \ln[f(x)]$. After simplifying $\ln[f(x)]$ by using properties of logarithms, we differentiate both sides of $\ln y = \ln[f(x)]$ with respect to x and then solve for y'. Logarithmic differentiation can also be used to differentiate $y = u^v$, where both u and v are functions of x.

Newton's method is the name given to the following formula which is used to provide a sequence of approximations to a roots of the equation $f(x) = 0$, for f a differentiable function:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad n = 1, 2, 3, \ldots$$

In most cases you might encounter, the approximations improve as n increases.

Because the derivative $f'(x)$ of a function $y = f(x)$ is itself a function, it can be successively differentiated to obtain the second derivative $f''(x)$, the third derivative $f'''(x)$, and other higher-order derivatives.

Review Problems

Problem numbers shown in color indicate problems suggested for use as a practice chapter test.

In Problems 1–30, differentiate.

1. $y = 3e^x + e^2 + e^{x^2} + x^{e^2}$ **2.** $f(w) = we^w + w^2$

3. $f(r) = \ln(r^2 + 5r)$ **4.** $y = e^{\ln x}$

5. $y = e^{x^2+4x+5}$ **6.** $f(t) = \log_6 \sqrt{t^2 + 1}$

7. $y = e^x(x^2 + 2)$ **8.** $y = 3^{5x^3}$

9. $y = \sqrt{(x-6)(x+5)(9-x)}$

10. $f(t) = e^{\sqrt{t}}$ **11.** $y = \dfrac{\ln x}{e^x}$

12. $y = \dfrac{e^x + e^{-x}}{x^2}$

13. $f(q) = \ln[(q+1)^2(q+2)^3]$

14. $y = (x+2)^3(x+1)^4(x-2)^2$

15. $y = 10^{2-7x}$ **16.** $y = (e + e^2)^0$

17. $y = \dfrac{4e^{3x}}{xe^{x-1}}$ **18.** $y = \dfrac{e^x}{\ln x}$

19. $y = \log_2(8x+5)^2$ **20.** $y = \ln\left(\dfrac{5}{x^2}\right)$

21. $f(l) = \ln(1 + l + l^2 + l^3)$

22. $y = x^{x^3}$

23. $y = (x+1)^{x+1}$ **24.** $y = \dfrac{1+e^x}{1-e^x}$

25. $\phi(t) = \ln(t\sqrt{4-t^2})$ **26.** $y = (x+3)^{\ln x}$

27. $y = \dfrac{(x^2+2)^{3/2}(x^2+9)^{4/9}}{(x^3+6x)^{4/11}}$

28. $y = \dfrac{\ln x}{\sqrt{x}}$ **29.** $y = (x^x)^x$ **30.** $y = x^{(x^x)}$

In Problems 31–34, evaluate y' at the given value of x.

31. $y = (x+1)\ln x^2$, $x = 1$

32. $y = \dfrac{e^{x^2+1}}{\sqrt{x^2+1}}$, $x = 1$

33. $y = e^{e+x\ln(1/x)}$, $x = e$

34. $y = \left[\dfrac{4^{3x}(x^3 - x + 1)^{1/5}}{(x^2+x+1)^4}\right]^{-2}$, $x = 0$

In Problems 35 and 36, find an equation of the tangent line to the curve at the point corresponding to the given value of x.

35. $y = 3e^x$, $x = \ln 2$ **36.** $y = x + x^2 \ln x$, $x = 1$

37. Find the y-intercept of the tangent line to the graph of $y = x(2^{2-x^2})$ at the point where $x = 1$.

38. If $w = 2^{x+1} + \ln(1 + x^2)$ and

$$x = \log_2(t^2 + 1) - e^{(t-1)^2}$$

find w and dw/dt when $t = 1$.

In Problems 39–42, find the indicated derivative at the given point.

39. $y = e^{x^2-4}$, y'', $(2, 1)$ **40.** $y = x^2e^x$, y''', $(1, e)$

41. $y = \ln(2x)$, y''', $(1, \ln 2)$ **42.** $y = x \ln x$, y'', $(1, 0)$

In Problems 43–46, find dy/dx.

43. $2xy + y^2 = 10$

44. $x^3y^3 = 3$

45. $\ln(xy^2) = xy$

46. $(\ln y)e^{y \ln x} = e^2$

In Problems 47 and 48, find d^2y/dx^2 at the given point.

47. $x + xy + y = 5$, $(2, 1)$

48. $xy + y^2 = 2$, $(-1, 2)$

49. If y is defined implicitly by $e^y = (y + 1)e^x$, determine both dy/dx and d^2y/dx^2 as explicit functions of y only.

50. If $\sqrt{x} + \sqrt{y} = 1$, find $\dfrac{d^2y}{dx^2}$.

51. Schizophrenia Several models have been used to analyze the length of stay in a hospital. For a particular group of schizophrenics, one such model is[9]

$$f(t) = 1 - (0.8e^{-0.01t} + 0.2e^{-0.0002t})$$

where $f(t)$ is the proportion of the group that was discharged at the end of t days of hospitalization. Determine the discharge rate (proportion discharged per day) at the end of t days.

52. Earthquakes According to Richter,[10] the number N of earthquakes of magnitude M or greater per unit of time is given by $\log N = A - bM$, where A and b are constants. He claims that

$$\log\left(-\frac{dN}{dM}\right) = A + \log\left(\frac{b}{q}\right) - bM$$

where $q = \log e$. Verify this statement.

53. If $f(x) = e^{9x^4+4x^3-36x}$, find all real zeros of $f'(x)$. Round your answers to two decimal places.

[9]Adapted from W. W. Eaton and G. A. Whitmore, "Length of Stay as a Stochastic Process: A General Approach and Application to Hospitalization for Schizophrenia," *Journal of Mathematical Sociology* 5 (1977), 273–92.

[10]C. F. Richter, *Elementary Seismology* (San Francisco: W. H. Freeman and Company, Publishers, 1958).

54. If $f(x) = \dfrac{x^5}{10} + \dfrac{x^4}{6} + \dfrac{2x^3}{3} + x^2 + 1$, find all zeros of $f''(x)$. Round your answers to two decimal places.

For the demand equations in Problems 55–57, determine whether demand is elastic, is inelastic, or has unit elasticity for the indicated value of q.

55. $p = \dfrac{500}{q}$; $q = 200$.

56. $p = 900 - q^2$; $q = 10$.

57. $p = 18 - 0.02q$; $q = 600$.

58. The demand equation for a product is

$$p = 30 - \sqrt{q}.$$

(a) Find the point elasticity of demand when $p = 10$.
(b) Verify that demand is inelastic if $0 < p < 10$.

59. The demand equation of a product is

$$q = \sqrt{2500 - p^2}.$$

Find the point elasticity of demand when $p = 30$. If the price of 30 decreases $\frac{2}{3}\%$, what is the approximate change in demand?

60. The demand equation for a product is

$$q = \sqrt{100 - p}, \quad \text{where } 0 < p < 100.$$

(a) Find all prices that correspond to elastic demand.
(b) Compute the point elasticity of demand when $p = 40$. Use your answer to estimate the percentage increase or decrease in demand when price is increased by 5% to $p = 42$.

61. The equation $x^3 - 2x - 2 = 0$ has a root between 1 and 2. Use Newton's method to estimate the root. Continue the approximation procedure until the difference of two successive approximations is less than 0.0001. Round your answer to four decimal places.

62. Find, to an accuracy of three decimal places, all real solutions of the equation $e^x = 3x$.

The line $y = b$ is a horizontal asymptote for the graph of a nonlinear function f if at least one of the following is true:

$$\lim_{x\to\infty} f(x) = b \quad \text{or} \quad \lim_{x\to-\infty} f(x) = b$$

In particular, a polynomial function has neither a horizontal nor a vertical asymptote. Moreover, a rational function whose numerator has degree greater than that of the denominator does not have a horizontal asymptote.

In applied work the importance of calculus in maximization and minimization problems can hardly be overstated. For example, in the area of economics, we can maximize profit or minimize cost. Some important relationships that are used in economics problems are the following:

$\bar{c} = \dfrac{c}{q}$	average cost per unit $= \dfrac{\text{total cost}}{\text{quantity}}$
$r = pq$	revenue = (price)(quantity)
$P = r - c$	profit = total revenue − total cost

Review Problems

Problem numbers shown in color indicate problems suggested for use as a practice chapter test.

In Problems 1–4, find horizontal and vertical asymptotes.

1. $y = \dfrac{3x^2}{x^2 - 16}$

2. $y = \dfrac{x+3}{9x - 3x^2}$

3. $y = \dfrac{5x^2 - 3}{(3x+2)^2}$

4. $y = \dfrac{4x+1}{3x-5} - \dfrac{3x+1}{2x-11}$

In Problems 5–8, find critical values.

5. $f(x) = \dfrac{7x^2}{2 - x^2}$

6. $f(x) = 8(x-1)^2(x+6)^4$

7. $f(x) = \dfrac{\sqrt[3]{x+1}}{3 - 4x}$

8. $f(x) = \dfrac{13xe^{-5x/6}}{6x+5}$

In Problems 9–12, find intervals on which the function is increasing or decreasing.

9. $f(x) = -\frac{5}{3}x^3 + 15x^2 + 35x + 10$

10. $f(x) = \dfrac{2x^2}{(x+1)^2}$

11. $f(x) = \dfrac{6x^4}{x^2 - 3}$

12. $f(x) = 9\sqrt[3]{3x^3 - 4x}$

In Problems 13–18, find intervals on which the function is concave up or concave down.

13. $f(x) = x^4 - x^3 - 14$

14. $f(x) = \dfrac{x-2}{x+2}$

15. $f(x) = \dfrac{1}{2x-1}$

16. $f(x) = x^3 + 2x^2 - 5x + 2$

17. $f(x) = (4x+1)^3(4x+9)$

18. $f(x) = (x^2 - x - 1)^2$

In Problems 19–24, test for relative extrema.

19. $f(x) = 2x^3 - 9x^2 + 12x + 7$

20. $f(x) = \dfrac{2x+1}{x^2}$

21. $f(x) = \dfrac{x^{10}}{10} + \dfrac{x^5}{5}$

22. $f(x) = \dfrac{x^2}{x^2 - 4}$

23. $f(x) = x^{2/3}(x+1)$

24. $f(x) = x^3(2x-1)^4$

In Problems 25–30, find the x-values where inflection points occur.

25. $y = x^5 - 5x^4 + 3x$

26. $y = \dfrac{x^2 + 2}{5x}$

27. $y = 4(3x-5)(x^4+2)$

28. $y = x^2 + 2\ln(-x)$

29. $y = \dfrac{x^2}{5e^x}$

30. $y = 6(x^2 - 4)^3$

In Problems 31–34, test for absolute extrema on the given interval.

31. $f(x) = 3x^4 - 4x^3$, $[0, 2]$

32. $f(x) = 2x^3 - 15x^2 + 36x$, $[0, 3]$

33. $f(x) = \dfrac{x}{(5x-6)^2}$, $[-2, 0]$

34. $f(x) = (x+1)^2(x-1)^{2/3}$, $[2, 3]$

35. Let $f(x) = (x^2 + 1)e^{-x}$

(a) Determine the values of x at which relative maxima and relative minima, if any, occur.

(b) Determine the interval(s) on which the graph of f is concave down, and find the coordinates of all points of inflection.

36. Let $f(x) = \frac{x}{x^2+1}$. It can be shown that

$$f'(x) = \frac{1 - x^2}{(x^2+1)^2}$$

(a) Determine whether the graph of f is symmetric about the x-axis, y-axis, or origin.

(b) Find the interval(s) on which f is increasing.

(c) Find the coordinates of all relative extrema of f.

(d) Determine $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.

(e) Sketch the graph of f.

(f) State the absolute minimum and absolute maximum values of $f(x)$ (if they exist).

In Problems 37–48, indicate intervals on which the function is increasing, decreasing, concave up, or concave down; indicate relative maximum points, relative minimum points, points of inflection, horizontal asymptotes, vertical asymptotes, *symmetry, and those intercepts that can be obtained conveniently and sketch the graph.*

37. $y = x^2 - 2x - 24$

38. $y = 2x^3 + 15x^2 + 36x + 9$

39. $y = x^3 - 12x + 20$

40. $y = x^4 - 4x^3 - 20x^2 + 150$

41. $y = x^3 + x$

42. $y = \dfrac{x+2}{x-3}$

43. $f(x) = \dfrac{100(x+5)}{x^2}$

44. $y = \dfrac{x^2-4}{x^2-1}$

45. $y = \dfrac{x}{(2x-1)^3}$

46. $y = 6x^{1/3}(2x-1)$

47. $f(x) = \dfrac{e^x + e^{-x}}{2}$

48. $f(x) = 1 + \ln(x^2)$

49. Are the following statements true or false?

(a) If $f'(x_0) = 0$, then f must have a relative extremum at x_0.

(b) Since the function $f(x) = 1/x$ is decreasing on the intervals $(-\infty, 0)$ and $(0, \infty)$, it is impossible to find x_1 and x_2 in the domain of f such that $x_1 < x_2$ and $f(x_1) < f(x_2)$.

(c) On the interval $(-1, 1]$, the function $f(x) = x^4$ has an absolute maximum and an absolute minimum.

(d) If $f''(x_0) = 0$, then $(x_0, f(x_0))$ must be a point of inflection.

(e) A function f defined on the interval $(-2, 2)$ with exactly one relative maximum must have an absolute maximum.

50. An important function in probability theory is the standard normal-density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

(a) Determine whether the graph of f is symmetric about the x-axis, y-axis, or origin.

(b) Find the intervals on which f is increasing and those on which it is decreasing.

(c) Find the coordinates of all relative extrema of f.

(d) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.

(e) Find the intervals on which the graph of f is concave up and those on which it is concave down.

(f) Find the coordinates of all points of inflection.

(g) Sketch the graph of f.

(h) Find all absolute extrema.

51. Marginal Cost If $c = q^3 - 6q^2 + 12q + 18$ is a total-cost function, for what values of q is marginal cost increasing?

52. Marginal Revenue If $r = 320q^{3/2} - 2q^2$ is the revenue function for a manufacturer's product, determine the intervals on which the marginal revenue function is increasing.

53. Revenue Function The demand equation for a manufacturer's product is

$$p = 150 - \frac{\sqrt{q}}{10} \quad \text{where } q > 0$$

Show that the graph of the revenue function is concave down wherever it is defined.

54. Contraception In a model of the effect of contraception on birthrate,[17] the equation,

$$R = f(x) = \frac{x}{4.4 - 3.4x} \qquad 0 \le x \le 1$$

gives the proportional reduction R in the birthrate as a function of the efficiency x of a contraception method. An efficiency of 0.2 (or 20%) means that the probability of becoming pregnant is 80% of the probability of becoming pregnant without the contraceptive. Find the reduction (as a percentage) when efficiency is (a) 0, (b) 0.5, and (c) 1. Find dR/dx and d^2R/dx^2, and sketch the graph of the equation.

55. Learning and Memory If you were to recite members of a category, such as four-legged animals, the words that you utter would probably occur in "chunks," with distinct pauses between such chunks. For example, you might say the following for the category of four-legged animals:

dog, cat, mouse, rat,
(pause)
horse, donkey, mule,
(pause)
cow, pig, goat, lamb,
etc.

The pauses may occur because one has to mentally search for subcategories (animals around the house, beasts of burden, farm animals, etc.).

The elapsed time between onsets of successive words is called *interresponse time*. A function has been used to analyze the length of time for pauses and the chunk size (number of words in a chunk).[18] This function f is such that

$$f(t) = \begin{cases} \text{the average number of words} \\ \text{that occur in succession with} \\ \text{interresponse times less than } t \end{cases}$$

The graph of f has a shape similar to that in Figure 13.72 and is best fit by a third-degree polynomial, such as

$$f(t) = At^3 + Bt^2 + Ct + D$$

[17] R. K. Leik and B. F. Meeker, *Mathematical Sociology* (Englewood Cliffs, NJ: Prentice-Hall, 1975).

[18] A. Graesser and G. Mandler, "Limited Processing Capacity Constrains the Storage of Unrelated Sets of Words and Retrieval from Natural Categories," *Human Learning and Memory* 4, no. 1 (1978), 86–100.

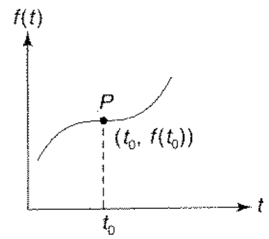

FIGURE 13.72 Diagram for Problem 55.

The point P has special meaning. It is such that the value t_0 separates interresponse times *within* chunks from those *between* chunks. Mathematically, P is a critical point that is also a point of inflection. Assume these two conditions, and show that (a) $t_0 = -B/(3A)$ and (b) $B^2 = 3AC$.

56. Market Penetration In a model for the market penetration of a new product, sales S of the product at time t are given by[19]

$$S = g(t) = \frac{m(p+q)^2}{p}\left[\frac{e^{-(p+q)t}}{\left(\frac{q}{p}e^{-(p+q)t}+1\right)^2}\right]$$

where p, q, and m are nonzero constants.

(a) Show that

$$\frac{dS}{dt} = \frac{\frac{m}{p}(p+q)^3 e^{-(p+q)t}\left[\frac{q}{p}e^{-(p+q)t} - 1\right]}{\left(\frac{q}{p}e^{-(p+q)t}+1\right)^3}$$

(b) Determine the value of t for which maximum sales occur. You may assume that S attains a maximum when $dS/dt = 0$.

In Problems 57–60, where appropriate, round your answers to two decimal places.

57. From the graph of $y = 4x^3 + 5.3x^2 - 7x + 3$, find the coordinates of all relative extrema.

58. From the graph of $f(x) = x^4 - 2x^3 + 3x - 1$, determine the absolute extrema of f over the interval $[-1, 1]$.

59. The graph of a function f has exactly one inflection point. If

$$f''(x) = \frac{x^3 + 3x + 2}{5x^2 - 2x + 4}$$

use the graph of f'' to determine the x-value of the inflection point of f.

[19] A. P. Hurter, Jr., A. H. Rubenstein, et al., "Market Penetration by New Innovations: The Technological Literature," *Technological Forecasting and Social Change* 11 (1978), 197–221.

60. Graph $y = \dfrac{4x - 2x^2}{x^3 + 4x + 4}$. From the graph, locate any horizontal or vertical asymptotes.

61. Maximization of Production A manufacturer determined that m employees on a certain production line will produce q units per month, where

$$q = 80m^2 - 0.1m^4$$

To obtain maximum monthly production, how many employees should be assigned to the production line?

62. Revenue The demand function for a manufacturer's product is given by $p = 100e^{-0.1q}$. For what value of q does the manufacturer maximize total revenue?

63. Revenue The demand function for a monopolist's product is

$$p = \sqrt{500 - q}$$

If the monopolist wants to produce at least 100 units, but not more than 200 units, how many units should be produced to maximize total revenue?

64. Average Cost If $c = 0.01q^2 + 5q + 100$ is a cost function, find the average-cost function. At what level of production q is there a minimum average cost?

65. Profit The demand function for a monopolist's product is

$$p = 400 - 2q$$

and the average cost per unit for producing q units is

$$\bar{c} = q + 160 + \frac{2000}{q}$$

where p and $\bar{c}$ are in dollars per unit. Find the maximum profit that the monopolist can achieve.

66. Container Design A rectangular box is to be made by cutting out equal squares from each corner of a piece of cardboard 10 in. by 16 in. and then folding up the sides. What must be the length of the side of the square cut out if the volume of the box is to be maximum?

67. Fencing A rectangular portion of a field is to be enclosed by a fence and divided equally into three parts by two fences parallel to one pair of the sides. If a total of 800 ft of fencing is to be used, find the dimensions that will maximize the fenced area.

68. Poster Design A rectangular poster having an area of 500 in^2 is to have a 4-in. margin at each side and at the bottom and a 6-in. margin at the top. The remainder of the poster is for printed matter. Find the dimensions of the poster so that the area for the printed matter is maximized.

69. Cost A furniture company makes personal-computer stands. For a certain model, the total cost (in thousands of dollars) when q *hundred* stands are produced is given by

$$c = 2q^3 - 9q^2 + 12q + 20$$

If $f(x) \geq 0$ and is continuous on $[a, b]$, then the definite integral may be used to find the area of the region bounded by $y = f(x)$ and the x-axis from $x = a$ to $x = b$. The definite integral may also be used to find areas of more complicated regions. In these situations, an element of area should be drawn in the region. This will allow you to set up the proper definite integral. In some situations vertical elements should be considered, whereas in others horizontal elements are more advantageous.

One application of finding areas involves consumers' surplus and producers' surplus. Suppose the market for a product is at equilibrium and (q_0, p_0) is the equilibrium point (the point of intersection of the supply and demand curves for the product). Then consumers' surplus, CS, corresponds to the area from $q = 0$ to $q = q_0$, bounded above by the demand curve and below by the line $p = p_0$. Thus,

$$\text{CS} = \int_0^{q_0} [f(q) - p_0]\, dq$$

where f is the demand function. Producers' surplus, PS, corresponds to the area from $q = 0$ to $q = q_0$, bounded above by the line $p = p_0$ and below by the supply curve. Therefore,

$$\text{PS} = \int_0^{q_0} [p_0 - g(q)]\, dq$$

where g is the supply function.

Review Problems

Problem numbers shown in color indicate problems suggested for use as a practice chapter test.

In Problems 1–40, determine the integrals.

1. $\int (x^3 + 2x - 7)\, dx$

2. $\int dx$

3. $\int_0^8 (\sqrt{2x} + 2x)\, dx$

4. $\int \frac{4}{5 - 3x}\, dx$

5. $\int \frac{6}{(x+5)^3}\, dx$

6. $\int_4^{12} (y - 8)^{501}\, dy$

7. $\int \frac{6x^2 - 12}{x^3 - 6x + 1}\, dx$

8. $\int_0^2 x e^{4 - x^2}\, dx$

9. $\int_0^1 \sqrt[3]{3t + 8}\, dt$

10. $\int \frac{4 - 2x}{7}\, dx$

11. $\int y(y+1)^2\, dy$

12. $\int_0^1 10^{-8}\, dx$

13. $\int \frac{\sqrt[4]{z} - \sqrt[3]{z}}{\sqrt{z}}\, dz$

14. $\int \frac{(0.5x - 0.1)^4}{0.4}\, dx$

15. $\int_1^3 \frac{2t^2}{3 + 2t^3}\, dt$

16. $\int \frac{4x^2 - x}{x}\, dx$

17. $\int x^2 \sqrt{3x^3 + 2}\, dx$

18. $\int (2x^3 + x)(x^4 + x^2)^{3/4}\, dx$

19. $\int (e^{2y} - e^{-2y})\, dy$

20. $\int \frac{8x}{3\sqrt[3]{7 - 2x^2}}\, dx$

21. $\int \left(\frac{1}{x} + \frac{2}{x^2}\right) dx$

22. $\int_0^2 \frac{3e^{3x}}{1 + e^{3x}}\, dx$

23. $\int_{-2}^1 10(y^4 - y + 1)\, dy$

24. $\int_7^{70} dx$

25. $\int_{\sqrt{3}}^2 7x\sqrt{4 - x^2}\, dx$

26. $\int_0^1 (2x + 1)(x^2 + x)^4\, dx$

27. $\int_0^1 \left[2x - \frac{1}{(x+1)^{2/3}}\right] dx$

28. $\int_3^{27} 3(\sqrt{3x} - 2x + 1)\, dx$

29. $\int \frac{\sqrt{t} - 3}{t^2}\, dt$

30. $\int \frac{2z^2}{z - 1}\, dz$

31. $\int_{-1}^0 \frac{x^2 + 4x - 1}{x + 2}\, dx$

32. $\int \frac{(x^2 + 4)^2}{x^2}\, dx$

33. $\int 9\sqrt{x}\sqrt{x^{3/2} + 1}\, dx$

34. $\int \frac{e^{\sqrt{5x}}}{\sqrt{3x}}\, dx$

35. $\int_1^e \frac{e^{\ln x}}{x^2}\, dx$

36. $\int \frac{6x^2 + 4}{e^{x^3 + 2x}}\, dx$

37. $\int \frac{(1 + e^{3x})^2}{e^{-3x}}\, dx$

38. $\int \frac{3}{e^{3x}(6 + e^{-3x})^2}\, dx$

39. $\int 3\sqrt{10^{3x}}\, dx$

40. $\int \frac{5x^3 + 15x^2 + 37x + 3}{x^2 + 3x + 7}\, dx$

In Problems 41 and 42, find y, subject to the given condition.

41. $y' = e^{2x} + 3, \quad y(0) = -\frac{1}{2}$

42. $y' = \frac{x + 3}{x}, \quad y(1) = 5$

In Problems 43–50, determine the area of the region bounded by the given curve, the x-axis, and the given lines.

43. $y = x^2 - 1, \quad x = 2$

44. $y = 4e^x, \quad x = 0, \quad x = 3$

45. $y = \sqrt{x + 4}, \quad x = 0$

46. $y = x^2 - x - 6, \quad x = -4, \quad x = 3$

47. $y = 5x - x^2$

48. $y = \sqrt[4]{x}, \quad x = 1, \quad x = 16$

49. $y = \frac{1}{x} + 3, \quad x = 1, \quad x = 3$

50. $y = x^3 - 1, \quad x = -1$

In Problems 51–58, find the area of the region bounded by the given curves.

51. $y^2 = 4x, \quad x = 0, \quad y = 2$

52. $y = 3x^2 - 5, \quad x = 0, \quad y = 4$

53. $y = x^2 + 4x - 5, \quad y = 0$

54. $y = 2x^2, \quad y = x^2 + 9$

55. $y = x^2 - 2x, \quad y = 12 - x^2$

56. $y = \sqrt{x}, \quad x = 0, \quad y = 3$

57. $y = \ln x, \quad x = 0, \quad y = 0, \quad y = 1$

58. $y = 2 - x, \quad y = x - 3, \quad y = 0, \quad y = 2$

59. Marginal Revenue If marginal revenue is given by

$$\frac{dr}{dq} = 100 - \frac{3}{2}\sqrt{2q}$$

determine the corresponding demand equation.

60. Marginal Cost If marginal cost is given by

$$\frac{dc}{dq} = q^2 + 7q + 6$$

and fixed costs are 2500, determine the total cost of producing six units. Assume that costs are in dollars.

61. Marginal Revenue A manufacturer's marginal-revenue function is

$$\frac{dr}{dq} = 275 - q - 0.3q^2$$

If r is in dollars, find the increase in the manufacturer's total revenue if production is increased from 10 to 20 units.

62. Marginal Cost A manufacturer's marginal-cost function is

$$\frac{dc}{dq} = \frac{1000}{\sqrt{3q + 70}}$$

If c is in dollars, determine the cost involved to increase production from 10 to 33 units.

63. Hospital Discharges For a group of hospitalized individuals, suppose the discharge rate is given by

$$f(t) = 0.008e^{-0.008t}$$

where $f(t)$ is the proportion discharged per day at the end of t days of hospitalization. What proportion of the group is discharged at the end of 100 days?

64. Business Expenses The total expenditures (in dollars) of a business over the next five years is given by

$$\int_0^5 4000e^{0.05t}\, dt$$

Evaluate the expenditures.

65. Find the area of the region between the curves $y = 9 - 2x$ and $y = x$ from $x = 0$ to $x = 4$.

66. Find the area of the region between the curves $y = x^2$ and $y = 4 - 3x$ from $x = -1$ to $x = 2$.

67. Consumers' and Producers' Surplus The demand equation for a product is

$$p = 0.01q^2 - 1.1q + 30$$

and the supply equation is

$$p = 0.01q^2 + 8$$

Determine consumers' surplus and producers' surplus when market equilibrium has been established.

68. Consumers' Surplus The demand equation for a product is

$$p = (q - 5)^2$$

and the supply equation is

$$p = q^2 + q + 3$$

where p (in thousands of dollars) is the price per 100 units when q hundred units are demanded or supplied. Determine consumers' surplus under market equilibrium.

69. Biology In a discussion of gene mutation,[16] the equation

$$\int_{q_0}^{q_n} \frac{dq}{q - \hat{q}} = -(u + v)\int_0^n dt$$

occurs, where u and v are gene mutation rates, the q's are gene frequencies, and n is the number of generations. Assume that all letters represent constants, except q and t. Integrate both sides and then use your result to show that

$$n = \frac{1}{u + v} \ln\left|\frac{q_0 - \hat{q}}{q_n - \hat{q}}\right|$$

70. Fluid Flow In studying the flow of a fluid in a tube of constant radius R, such as blood flow in portions of the body, one can think of the tube as consisting of concentric tubes of radius r, where $0 \le r \le R$. The velocity v of the fluid is a function of r and is given by[17]

$$v = \frac{(P_1 - P_2)(R^2 - r^2)}{4\eta l}$$

[16] W. B. Mather, *Principles of Quantitative Genetics* (Minneapolis: Burgess Publishing Company, 1964).

[17] R. W. Stacy et al., *Essentials of Biological and Medical Physics* (New York: McGraw-Hill Book Company, 1955).

Review Problems

Problem numbers shown in color indicate problems suggested for use as a practice chapter test.

In Problems 1–22, determine the integrals.

1. $\int x \ln x \, dx$
2. $\int \frac{1}{\sqrt{4x^2+1}} \, dx$
3. $\int_0^2 \sqrt{4x^2+9} \, dx$
4. $\int \frac{16x}{3-4x} \, dx$
5. $\int \frac{15x-2}{(3x+1)(x-2)} \, dx$
6. $\int_e^{e^2} \frac{1}{x \ln x} \, dx$
7. $\int \frac{dx}{x(x+2)^2}$
8. $\int \frac{dx}{x^2-1}$
9. $\int \frac{dx}{x^2\sqrt{9-16x^2}}$
10. $\int x^{1/3} \ln \sqrt{x} \, dx$
11. $\int \frac{9\, dx}{x^2-9}$
12. $\int \frac{x}{\sqrt{2+5x}} \, dx$
13. $\int 49xe^{7x} \, dx$
14. $\int \frac{dx}{2+3e^{4x}}$
15. $\int \frac{dx}{2x \ln 2x}$
16. $\int \frac{dx}{x(2+x)}$
17. $\int \frac{2x}{3+2x} \, dx$
18. $\int \frac{dx}{x^2\sqrt{4x^2-9}}$

[16] 19. $\int \frac{5x^2+2}{x^3+x} \, dx$

[16] 20. $\int \frac{3x^3+5x^2+4x+3}{x^4+x^3+x^2} \, dx$

[17] 21. $\int \frac{\ln(x+1)}{\sqrt{x+1}} \, dx$

[17] 22. $\int (\ln x)^3 \, dx$

23. Find the average value of $f(x) = 3x^2 + 2x$ over the interval $[2, 4]$.

24. Find the average value of $f(t) = t^2e^t$ over the interval $[0, 1]$.

In Problems 25 and 26, solve the differential equations.

25. $y' = 3x^2y + 2xy \qquad y > 0$

26. $y' - 2xe^{x^2-y+3} = 0 \qquad y(0) = 3$

In Problems 27–30, determine the improper integrals if they exist.[18] *Indicate those that are divergent.*

27. $\int_3^{\infty} \frac{1}{x^3} \, dx$
28. $\int_{-\infty}^{0} e^{2x} \, dx$
29. $\int_1^{\infty} \frac{1}{2x} \, dx$
30. $\int_{-\infty}^{\infty} xe^{1-x^2} \, dx$

31. **Population** The population of a city in 1985 was 100,000 and in 2000 was 120,000. Assuming exponential growth, project the population in 2015.

32. **Population** The population of a city doubles every 10 years due to exponential growth. At a certain time, the population is 40,000. Find an expression for the number of people N at time t years later. Give your answer in terms of ln 2.

33. **Radioactive** If 95% of a radioactive substance remains after 100 years, find the decay constant and, to the nearest percent, give the percentage of the original amount present after 200 years.

34. **Medicine** Suppose q is the amount of penicillin in the body at time t, and let q_0 be the amount at $t = 0$. Assume that the rate of change of q with respect to t is proportional to q and that q decreases as t increases. Then we have $dq/dt = -kq$, where $k > 0$. Solve for q. What percentage of the original amount present is there when $t = 5/k$?

35. **Biology** Two organisms are initially placed in a medium and begin to multiply. The number N of organisms that are present after t days is recorded on a graph with the horizontal axis labeled t and the vertical axis labeled N. It is observed that the points lie on a logistic curve. The number of organisms present after 6 days is 300, and beyond 10 days the number approaches a limit of 450. Find the logistic equation.

36. **College Enrollment** A college believes that enrollment follows logistic growth. Last year enrollment was 1000, and this year it is 1100. If the college can accommodate a maximum of 2000 students, what is the anticipated enrollment next year? Give your answer to the nearest hundred.

37. **Time of Murder** A coroner is called in on a murder case. He arrives at 6:00 P.M. and finds that the victim's temperature is 35°C. One hour later the body temperature is 34°C. The temperature of the room is 25°C. About what time was the murder committed? (Assume that normal body temperature is 37°C.)

38. **Annuity** Find the present value, to the nearest dollar, of a continuous annuity at an annual rate of 6% for 12 years if the payment at time t is at the annual rate of $f(t) = 10t$ dollars.

[19] 39. **Hospital Discharges** For a group of hospitalized individuals, suppose the proportion that has been discharged at the end of days t is given by

$$\int_0^t f(x)\,dx$$

where $f(x) = 0.008e^{-0.01x} + 0.00004e^{-0.0002x}$. Evaluate

$$\int_0^{\infty} f(x)\,dx$$

[16] Problems 19 and 20 refers to Section 15.2.

[17] Problems 21 and 22 refer to Section 15.1.

[18] Problems 27–30 refer to Section 15.7.

[19] Problems 39 and 40 refer to Section 15.7.

[19] 40.

[20] J. I

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The test states that if (x_0, y_0) is a critical point of f and

$$D(x, y) = f_{xx}(x, y) f_{yy}(x, y) - [f_{xy}(x, y)]^2$$

then

1. if $D(x_0, y_0) > 0$ and $f_{xx}(x_0, y_0) < 0$, f has a relative maximum at (x_0, y_0);
2. if $D(x_0, y_0) > 0$ and $f_{xx}(x_0, y_0) > 0$, f has a relative minimum at (x_0, y_0);
3. if $D(x_0, y_0) < 0$, f has neither a relative maximum nor a relative minimum at (x_0, y_0);
4. if $D(x_0, y_0) = 0$, no conclusion about an extremum at (x_0, y_0) can be drawn, and further analysis is required.

To find critical points of a function of several variables, subject to a constraint, we may use the method of Lagrange multipliers. For example, to find the critical points of $f(x, y, z)$, subject to the constraint $g(x, y, z) = 0$, we first form the function

$$F(x, y, z, \lambda) = f(x, y, z) - \lambda g(x, y, z)$$

By solving the system

$$F_x = 0 \quad F_y = 0 \quad F_z = 0 \quad F_\lambda = 0$$

we obtain the critical points of F. If $(x_0, y_0, z_0, \lambda_0)$ is such a critical point, then (x_0, y_0, z_0) is a critical point of f, subject to the constraint. It is important to write the constraint in the form $g(x, y, z) = 0$. For example, if the constraint is $2x + 3y - z = 4$, then $g(x, y, z) = 2x + 3y - z - 4$ [or $g(x, y, z) = 4 - 2x - 3y + z$]. If $f(x, y, z)$ is subject to two constraints, $g_1(x, y, z) = 0$ and $g_2(x, y, z) = 0$, then we would form the function $F = f - \lambda_1 g_1 - \lambda_2 g_2$ and solve the system

$$F_x = 0 \quad F_y = 0 \quad F_z = 0 \quad F_{\lambda_1} = 0 \quad F_{\lambda_2} = 0$$

Sometimes two variables, say, x and y, may be related in such a way that the relationship is approximately linear. When the data points (x_i, y_i), where $i = 1, 2, 3, \ldots, n$, are given to us, we can fit a straight line that approximates them. Such a line is the linear-regression line (or least squares line) of y on x and is given by

$$\hat{y} = \hat{a} + \hat{b}x$$

where

$$\hat{a} = \frac{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} x_i y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

and

$$\hat{b} = \frac{n\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

The $\hat{y}$-values can be used to predict y-values for given values of x.

When working with functions of several variables, we can consider their multiple integrals. These are determined by successive integration. For example, the double integral

$$\int_1^2 \int_0^y (x + y)\, dx\, dy$$

is determined by first treating y as a constant and integrating $x + y$ with respect to x. After evaluating between the limits 0 and y, we integrate that result with respect to y from $y = 1$ to $y = 2$. Thus,

$$\int_1^2 \int_0^y (x + y)\, dx\, dy = \int_1^2 \left[\int_0^y (x + y)\, dx\right] dy$$

Triple integrals involve functions of three variables and are also evaluated by successive integration.

Review Problems

Problem numbers shown in color indicate problems suggested for use as a practice chapter test.

In Problems 1–4, sketch the given surfaces.

1. $2x + 3y + z = 9$ **2.** $z = x$

3. $z = y^2$ **4.** $x^2 + z^2 = 1$

In Problems 5–16, find the indicated partial derivatives.

5. $f(x, y) = 4x^2 + 6xy + y^2 - 1$; $f_x(x, y), f_y(x, y)$

6. $P = l^3 + k^3 - lk$; $\partial P/\partial l, \partial P/\partial k$

7. $z = \dfrac{x}{x + y}$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

8. $f(p_A, p_B) = 2(p_A - 20) + 3(p_B - 30)$; $f_{p_A}(p_A, p_B)$

9. $f(x, y) = \ln\sqrt{x^2 + y^2}$; $\dfrac{\partial}{\partial y}[f(x, y)]$

10. $w = \dfrac{x}{\sqrt{x^2 + y^2}}$; $\dfrac{\partial w}{\partial y}$

11. $w = e^{x^2 yz}$; $w_{xy}(x, y, z)$

12. $f(x, y) = xy\ln(xy)$; $f_{xy}(x, y)$

13. $f(x, y, z) = (x + y)(y + z^2)$; $\frac{\partial^2}{\partial z^2}[f(x, y, z)]$

14. $z = (x^2 - y)(y^2 - 2xy)$; $\partial^2 z/\partial y^2$

15. $w = e^{x+y+z} \ln xyz$; $\partial w/\partial y, \partial^2 w/\partial z\, \partial x$

16. $P = 100l^{0.11}k^{0.89}$; $\partial^2 P/\partial k \partial l$

17. If $f(x, y, z) = \frac{x + y}{xz}$, find $f_{xyz}(2, 7, 4)$.

18. If $f(x, y, z) = (6x + 1)e^{y^2 \ln(z+1)}$, find $f_{xyz}(0, 1, 0)$.

[27]**19.** If $w = x^2 + 2xy + 3y^2$, $x = e^r$, and $y = \ln(r + s)$, find $\partial w/\partial r$ and $\partial w/\partial s$.

[27]**20.** If $z = \ln(x/y) + e^y - xy$, $x = r^2 s^2$, and $y = r + s$, find $\partial z/\partial s$.

[27]**21.** If $x^2 + 2xy - 2z^2 + xz + 2 = 0$, find $\partial z/\partial x$.

[27]**22.** If $z^2 + \ln(yz) + \ln z + x + z = 0$, find $\partial z/\partial y$.

23. Production Function If a manufacturer's production function is defined by $P = 20l^{0.7}k^{0.3}$, determine the marginal productivity functions.

24. Joint-Cost Function A manufacturer's cost for producing x units of product X and y units of product Y is given by

$$c = 3x + 0.05xy + 9y + 500$$

determine the (partial) marginal cost with respect to x when $x = 50$ and $y = 100$.

25. Competitive/Complementary Products If $q_A = 200 - 3p_A + p_B$ and $q_B = 50 - 5p_B + p_A$, where q_A and q_B are the number of units demanded of products A and B, respectively, and p_A and p_B are their respective prices per unit, determine whether A and B are competitive or complementary products.

26. Innovation For industry, the following model describes the rate α (a Greek letter read "alpha") at which an innovation substitutes for an established process:[28]

$$\alpha = Z + 0.530P - 0.027S$$

Here, Z is a constant that depends on the particular industry, P is an index of profitability of the innovation, and S is an index of the extent of the investment necessary to make use of the innovation. Find $\partial\alpha/\partial P$ and $\partial\alpha/\partial S$.

27. Examine $f(x, y) = x^2 + 2y^2 - 2xy - 4y + 3$ for relative extrema.

28. Examine $f(w, z) = 2w^3 + 2z^3 - 6wz + 7$ for relative extrema.

29. Minimizing Material An open-top rectangular cardboard box is to have a volume of 32 cubic feet. Find the dimensions of the box so that the amount of cardboard used is minimized.

30. The function

$$f(x, y) = ax^2 + by^2 + cxy - 10x - 20y$$

has a critical point at $(x, y) = (1, 2)$, and the second-derivative test is inconclusive at this point. Determine the values of the constants a, b, and c.

31. Maximizing Profit A dairy produces two types of cheese, A and B, at constant average costs of 50 cents and 60 cents per pound, respectively. When the selling price per pound of A is p_A cents and of B is p_B cents, the demands (in pounds) for A and B, are, respectively,

$$q_A = 250(p_B - p_A)$$

and

$$q_B = 32{,}000 + 250(p_A - 2p_B)$$

Find the selling prices that yield a relative maximum profit. Verify that the profit has a relative maximum at these prices.

32. Find all critical points of $f(x, y, z) = x^2yz$, subject to the condition that

$$x + 3y - 2z - 80 = 0 \ (xyz \neq 0)$$

33. Find all critical points of $f(x, y, z) = x^2 + y^2 + z^2$, subject to the constraint $3x + 2y + z = 14$.

34. Surviving Infection In an experiment,[29] a group of fish was injected with living bacteria. Of those fish maintained at 28°C, the percentage p that survived the infection t hours after the injection is given in the following table:

t	8	10	18	20	48
p	82	79	78	78	64

Find the linear-regression line of p on t.

[27]Problems 19–22 refer to Section 17.4 or Section 17.6.

[28]A. P. Hurter, Jr., A. H. Rubenstein, et al., "Market Penetration by New Innovations: The Technological Literature," *Technological Forecasting and Social Change* 11 (1978), 197–221.

[29]J. B. Covert and W. W. Reynolds, "Survival Value of Fever in Fish," *Nature* 267, no. 5606 (1977), 43–45.

APPLICATIONS IN PRACTICE 10.5

1. $0 < x < 4$

EXERCISE 10.5 (page 530)

1. $(-\infty, -1), (4, \infty)$ **3.** $[3, 5]$ **5.** $\left(-\frac{7}{2}, -2\right)$
7. No solution **9.** $(-\infty, -6], [-2, 3]$
11. $(-\infty, -4), (0, 5)$ **13.** $[0, \infty)$ **15.** $(-3, 0), (1, \infty)$
17. $(-\infty, -3), (0, 3)$ **19.** $(1, \infty)$
21. $(-\infty, -5), [-2, 1), [3, \infty)$ **23.** $(-5, -1)$
25. $(-\infty, -1 - \sqrt{3}], [-1 + \sqrt{3}, \infty)$
27. Between 50 and 65 inclusive **29.** 17 in. by 17 in.
31. $(-\infty, -7.72]$ **33.** $(-\infty, -0.5), (0.667, \infty)$

REVIEW PROBLEMS—CHAPTER 10 (page 532)

1. -5 **3.** 2 **5.** x **7.** $-\frac{8}{3}$ **9.** 0 **11.** $\frac{5}{9}$
13. Does not exist **15.** -1 **17.** $\frac{1}{9}$ **19.** $-\infty$
21. ∞ **23.** $-\infty$ **25.** 1 **27.** $-\infty$ **29.** 8
31. 23 **33. (a)** \$5034.38; **(b)** \$1241.46 **35.** 6.18%
37. $\frac{\ln 2}{0.065}$
41. Continuous everywhere; f is a polynomial function.
43. $x = -3$ **45.** None **47.** $x = -4, 1$
49. $x = -2$ **51.** $(-\infty, -6), (2, \infty)$
53. $[2, \infty), x = 0$ **55.** $(-\infty, -5), (-1, 1)$
57. $(-\infty, -4), [-3, 0], (2, \infty)$ **59.** 1.00
61. 0 **63.** $[2.00, \infty)$

MATHEMATICAL SNAPSHOT—CHAPTER 10 (page 534)

1. 17%
3. An exponential model assumes a fixed repayment rate.

APPLICATIONS IN PRACTICE 11.1

1. $\frac{dH}{dt} = 40 - 32t$

EXERCISE 11.1 (page 544)

1. (a)

x-value of Q	3	2.5	2.2	2.1	2.01	2.001
m_{PQ}	19	15.25	13.24	12.61	12.0601	12.0060

(b) We estimate that $m_{\tan} = 12$.
3. 1 **5.** 3 **7.** -4 **9.** 0 **11.** $2x + 4$
13. $4q + 5$ **15.** $-\frac{6}{x^2}$ **17.** $\frac{1}{2\sqrt{x+2}}$ **19.** -4
21. 0 **23.** $y = x + 4$ **25.** $y = -3x - 7$
27. $y = -3x + 9$ **29.** $\frac{r}{r_L - r - \frac{dC}{dD}}$
31. $-3.000, 13.445$ **33.** $-5.120, 0.038$

35. For the x-values of the points where the tangent to the graph of f is horizontal, the corresponding values of $f'(x)$ are 0. This is expected because the slope of a horizontal line is zero and the derivative gives the slope of the tangent line.

APPLICATIONS IN PRACTICE 11.2

1. $50 - 0.6q$

EXERCISE 11.2 (page 551)

1. 0 **3.** $6x^5$ **5.** $80x^{79}$ **7.** $18x$ **9.** $20w^4$
11. $\frac{8}{3}x^3$ **13.** $\frac{7}{25}t^6$ **15.** 1 **17.** $8x - 2$
19. $4p^3 - 9p^2$ **21.** $-8x^7 + 5x^4$
23. $-39x^2 + 28x - 2$ **25.** $-8x^3$ **27.** $-\frac{4}{3}x^3$
29. $16x^3 + 3x^2 - 9x + 8$ **31.** $\frac{6}{5}x^3 + 7x^2$ **33.** $\frac{7}{2}x^{5/2}$
35. $\frac{3}{4}x^{-1/4} + \frac{10}{3}x^{2/3}$ **37.** $\frac{11}{2}x^{-1/2}$ or $\frac{11}{2\sqrt{x}}$ **39.** $2r^{-2/3}$
41. $-4x^{-5}$ **43.** $-3x^{-4} - 5x^{-6} + 12x^{-7}$
45. $-x^{-2}$ or $-\frac{1}{x^2}$ **47.** $-40x^{-6}$ **49.** $-4x^{-4}$
51. $-\frac{1}{2}t^{-2}$ **53.** $\frac{1}{7} - 7x^{-2}$ **55.** $-3x^{-2/3} - 2x^{-7/5}$
57. $-\frac{1}{5}x^{-6/5}$ **59.** $-x^{-3/2}$ **61.** $\frac{5}{2}x^{3/2}$
63. $9x^2 - 20x + 7$ **65.** $45x^4$
67. $\frac{1}{3}x^{-2/3} - \frac{10}{3}x^{-5/3} = \frac{1}{3}x^{-5/3}(x - 10)$ **69.** $8q + \frac{4}{q^2}$
71. $2x + 4$ **73.** 1 **75.** $4, 16, -14$ **77.** $0, 0, 0$
79. $y = 13x + 2$ **81.** $y = -4x + 6$
83. $y = x + 3$ **85.** $(0, 0), \left(\frac{5}{3}, \frac{125}{54}\right)$ **87.** $(3, -3)$
89. 0 **91.** The tangent line is $y = 9x - 16$.

APPLICATIONS IN PRACTICE 11.3

1. 2.5 units **2.** $\frac{dy}{dt} = 16 - 32t; \left.\frac{dy}{dt}\right|_{t=0.5} = 0$ feet/s
When $t = 0.5$ the object reaches its maximum height.
3. 1.2 and 120%

EXERCISE 11.3 (page 561)

1.

Δt	1	0.5	0.2	0.1	0.01	0.001
$\Delta s/\Delta t$	9	8	7.4	7.2	7.02	7.002

We estimate the velocity when $t = 1$ to be 7.0000 m/s.
With differentiation the velocity is 7 m/s.
3. (a) 4 m; **(b)** 5.5 m/s; **(c)** 5 m/s
5. (a) 8 m; **(b)** 6.1208 m/s; **(c)** 6 m/s
7. (a) 2 m; **(b)** 10.261 m/s; **(c)** 9 m/s
9. $\frac{dy}{dx} = \frac{25}{2}x^{3/2}$; 337.50 **11.** 0.27
13. $dc/dq = 10$; 10 **15.** $dc/dq = 0.6q + 2$; 3.8
17. $dc/dq = 2q + 50$; 80, 82, 84
19. $dc/dq = 0.02q + 5$; 6, 7

39. $(8x-1)^3[4(2x+1)^3(2)]+(2x+1)^4[3(8x-1)^2(8)]$
$=16(8x-1)^2(2x+1)^3(7x+1)$

41. $10\left(\dfrac{x-7}{x+4}\right)^9\left[\dfrac{(x+4)(1)-(x-7)(1)}{(x+4)^2}\right]$
$=\dfrac{110(x-7)^9}{(x+4)^{11}}$

43. $\dfrac{1}{2}\left(\dfrac{x-2}{x+3}\right)^{-1/2}\left[\dfrac{(x+3)(1)-(x-2)(1)}{(x+3)^2}\right]$
$=\dfrac{5}{2(x+3)^2}\left(\dfrac{x-2}{x+3}\right)^{-1/2}$

45. $\dfrac{(x^2+4)^3(2)-(2x-5)[3(x^2+4)^2(2x)]}{(x^2+4)^6}$
$=\dfrac{-2(5x^2-15x-4)}{(x^2+4)^4}$

47. $\dfrac{(3x-1)^3[40(8x-1)^4]-(8x-1)^5[9(3x-1)^2]}{(3x-1)^6}$
$=\dfrac{(8x-1)^4(48x-31)}{(3x-1)^4}$

49. $6\{(5x^2+2)[2x^3(x^4+5)^{-1/2}]+(x^4+5)^{1/2}(10x)\}$
$=12x(x^4+5)^{-1/2}(10x^4+2x^2+25)$

51. $8+\dfrac{5}{(t+4)^2}-(8t-7)=15-8t+\dfrac{5}{(t+4)^2}$

53. $\dfrac{(x^2-7)^4[(2x+1)(2)(3x-5)(3)+(3x-5)^2(2)]-(2x+1)(3x-5)^2[4(x^2-7)^3(2x)]}{(x^2-7)^8}$

55. 0 **57.** 0 **59.** $y=4x-11$

61. $y=-\dfrac{1}{6}x+\dfrac{5}{3}$ **63.** 96% **65.** 20 **67.** ≈ 13.99

69. **(a)** $-\dfrac{q}{\sqrt{q^2+20}}$; **(b)** $-\dfrac{q}{100\sqrt{q^2+20}-q^2-20}$;
(c) $100-\dfrac{q^2}{\sqrt{q^2+20}}-\sqrt{q^2+20}$

71. -481.5 **73.** $\dfrac{dc}{dq}=\dfrac{5q(q^2+6)}{(q^2+3)^{3/2}}$ **75.** $48\pi(10)^{-19}$

77. **(a)** $-0.001416x^3+0.01356x^2+1.696x-34.9$, -136.188; **(b)** -0.008 **79.** -4 **81.** 40

83. 86,111.37

REVIEW PROBLEMS—CHAPTER 11 (page 585)

1. $-2x$ **3.** $\dfrac{\sqrt{3}}{2\sqrt{x}}$ **5.** 0

7. $28x^3-18x^2+10x=2x(14x^2-9x+5)$

9. $4s^3+4s=4s(s^2+1)$ **11.** $\dfrac{2x}{5}$

13. $(x^2+6x)(3x^2-12x)+(x^3-6x^2+4)(2x+6)$
$=5x^4-108x^2+8x+24$

15. $100(2x^2+4x)^{99}(4x+4)$
$=400(x+1)(2x^2+4x)^{99}$

17. $-\dfrac{6}{(2x+1)^2}$

19. $(8+2x)(4)(x^2+1)^3(2x)+(x^2+1)^4(2)$
$=2(x^2+1)^3(9x^2+32x+1)$

21. $\dfrac{(z^2+4)(2z)-(z^2-1)(2z)}{(z^2+4)^2}=\dfrac{10z}{(z^2+4)^2}$

23. $\dfrac{4}{3}(4x-1)^{-2/3}$

25. $-\dfrac{1}{2}(1-x)^{-3/2}(-1)=\dfrac{1}{2}(1-x)^{-3/2}$

27. $(x-6)^4[3(x+5)^2]+(x+5)^3[4(x-6)^3]$
$=(x-6)^3(x+5)^2(7x+2)$

29. $\dfrac{(x+6)(5)-(5x-4)(1)}{(x+6)^2}=\dfrac{34}{(x+6)^2}$

31. $2\left(-\dfrac{3}{8}\right)x^{-11/8}+\left(-\dfrac{3}{8}\right)(2x)^{-11/8}(2)$
$=-\dfrac{3}{4}(1+2^{-11/8})x^{-11/8}$

33. $\dfrac{\sqrt{x^2+5}(2x)-(x^2+6)(1/2)(x^2+5)^{-1/2}(2x)}{x^2+5}$
$=\dfrac{x(x^2+4)}{(x^2+5)^{3/2}}$

35. $\left(\dfrac{3}{5}\right)(x^3+6x^2+9)^{-2/5}(3x^2+12x)$
$=\dfrac{9}{5}x(x+4)(x^3+6x^2+9)^{-2/5}$

37. $7(1-2z)$ **39.** $y=-4x+3$

41. $y=\dfrac{1}{12}x+\dfrac{4}{3}$ **43.** $\dfrac{5}{7}\approx 0.714$; 71.4%

45. $dr/dq=20-0.2q$ **47.** 0.569, 0.431

49. $dr/dq=450-q$

51. $dc/dq=0.125+0.00878q$; 0.7396

53. 84 eggs/mm **55.** **(a)** $\dfrac{4}{3}$; **(b)** $\dfrac{1}{24}$ **57.** 8π ft^3/ft

59. $4q-\dfrac{10{,}000}{q^2}$ **61.** **(a)** 240; **(b)** $\dfrac{1}{100}$;
(c) no, since $dr/dm < 300$ when $m=80$ **63.** 0.305

65. -0.32

MATHEMATICAL SNAPSHOT—CHAPTER 11 (page 588)

1. The slope is greater—above 0.9. More is spent; less is saved.
3. Spend \$705, save \$295 **5.** Answers may vary.

APPLICATIONS IN PRACTICE 12.1

1. $\dfrac{dq}{dp}=\dfrac{12p}{3p^2+4}$ **2.** $\dfrac{dR}{dI}=\dfrac{1}{I\ln 10}$

EXERCISE 12.1 (page 595)

1. $\dfrac{4}{x}$ **3.** $\dfrac{3}{3x-7}$ **5.** $\dfrac{2}{x}$ **7.** $-\dfrac{2x}{1-x^2}$

9. $\dfrac{6p^2+3}{2p^3+3p}=\dfrac{3(2p^2+1)}{p(2p^2+3)}$

11. $t\left(\dfrac{1}{t}\right)+(\ln t)=1+\ln t$

13. $\dfrac{2x^3}{2x+5}+3x^2\ln(2x+5)$ **15.** $\dfrac{8}{(\ln 3)(8x-1)}$

17. $2x\left[1+\dfrac{1}{(\ln 2)(x^2+4)}\right]$

19. $\dfrac{z\left(\dfrac{1}{z}\right)-(\ln z)(1)}{z^2}=\dfrac{1-\ln z}{z^2}$

EXERCISE 12.6 (page 622)

1. 0.25410 **3.** 1.32472 **5.** −0.68233 **7.** 0.33767
9. 1.90785 **11.** 4.141 **13.** −4.99 and 1.94
15. 13.33 **17.** 2.880 **19.** 3.45

APPLICATIONS IN PRACTICE 12.7

1. $\frac{d^2h}{dt^2} = -32$ feet/sec² (*Note*: Negative values indicate the downward direction.)
2. $c''(3) = 14$ dollars/unit²

EXERCISE 12.7 (page 626)

1. 24 **3.** 0 **5.** e^x **7.** $3 + 2\ln x$ **9.** $-\frac{10}{p^6}$
11. $-\frac{1}{4(9-r)^{3/2}}$ **13.** $\frac{8}{(2x+3)^3}$ **15.** $\frac{4}{(x-1)^3}$
17. $-\left[\frac{1}{x^2} + \frac{1}{(x+6)^2}\right]$ **19.** $e^z(z^2 + 4z + 2)$
21. 32 **23.** $-\frac{1}{y^3}$ **25.** $-\frac{4}{y^3}$ **27.** $\frac{1}{8x^{3/2}}$
29. $\frac{2(y-1)}{(1+x)^2}$ **31.** $\frac{y}{(1-y)^3}$ **33.** $-\frac{16}{125}$
35. $300(5x-3)^2$ **37.** 0.6 **39.** ±1
41. −4.99 and 1.94

REVIEW PROBLEMS—CHAPTER 12 (page 628)

1. $3e^x + 0 + e^{x^2}(2x) + (e^2)x^{e^2-1} = 3e^x + 2xe^{x^2} + e^2x^{e^2-1}$
3. $\frac{1}{r^2+5r}(2r+5) = \frac{2r+5}{r(r+5)}$
5. $e^{x^2+4x+5}(2x+4) = 2(x+2)e^{x^2+4x+5}$
7. $e^x(2x) + (x^2+2)e^x = e^x(x^2+2x+2)$
9. $\frac{\sqrt{(x-6)(x+5)(9-x)}}{2}\left[\frac{1}{x-6} + \frac{1}{x+5} + \frac{1}{x-9}\right]$
11. $\frac{e^x\left(\frac{1}{x}\right) - (\ln x)(e^x)}{e^{2x}} = \frac{1 - x\ln x}{xe^x}$
13. $\frac{2}{q+1} + \frac{3}{q+2}$ **15.** $-7(\ln 10)10^{2-7x}$
17. $\frac{4e^{2x+1}(2x-1)}{x^2}$ **19.** $\frac{16}{(8x+5)\ln 2}$
21. $\frac{1+2l+3l^2}{1+l+l^2+l^3}$ **23.** $(x+1)^{x+1}[1+\ln(x+1)]$
25. $\frac{1}{t} + \frac{1}{2}\cdot\frac{1}{4-t^2}\cdot(-2t) = \frac{1}{t} - \frac{t}{4-t^2}$
27. $y\left[\frac{3}{2}\left(\frac{1}{x^2+2}\right)(2x) + \frac{4}{9}\left(\frac{1}{x^2+9}\right)(2x) - \frac{4}{11}\left(\frac{1}{x^3+6x}\right)(3x^2+6)\right]$
$= y\left[\frac{3x}{x^2+2} + \frac{8x}{9(x^2+9)} - \frac{12(x^2+2)}{11(x^3+6x)}\right]$,
where y is as given in the problem
29. $(x^x)^x(x + 2x\ln x)$ **31.** 4 **33.** −2
35. $y = 6x + 6(1 - \ln 2)$ or $y = 6x + 6 - \ln 64$

37. $(0, 4\ln 2)$ **39.** 18 **41.** 2 **43.** $-\frac{y}{x+y}$
45. $\frac{xy^2 - y}{2x - x^2y}$ **47.** $\frac{4}{9}$
49. $\frac{dy}{dx} = \frac{y+1}{y}; \frac{d^2y}{dx^2} = -\frac{y+1}{y^3}$
51. $f'(t) = 0.008e^{-0.01t} + 0.00004e^{-0.0002t}$ **53.** 0.90
55. $\eta = -1$, unit elasticity
57. $\eta = -0.5$, demand is inelastic
59. $-\frac{9}{16}$, $\approx \frac{3}{8}$% increase **61.** 1.7693

MATHEMATICAL SNAPSHOT—CHAPTER 12 (page 630)

1. Figure 12.11 shows that the population reaches its final size in about 45 days.
3. The tangent line will not coincide exactly with the curve in the first place. Smaller time steps could reduce the error.

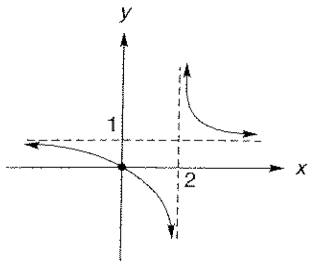
APPLICATIONS IN PRACTICE 13.1

1. There is a relative maximum when $q = 2$, and a relative minimum when $q = 5$.
2. The drug is at its greatest concentration 2 hours after injection.

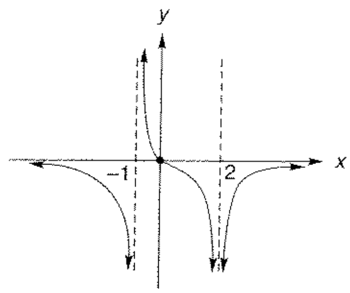
EXERCISE 13.1 (page 642)

1. Dec. on $(-\infty, -1)$ and $(3, \infty)$; inc. on $(-1, 3)$; rel. min. $(-1, -1)$; rel. max. $(3, 4)$
3. Dec. on $(-\infty, -2)$ and $(0, 2)$; inc. on $(-2, 0)$ and $(2, \infty)$; rel. min. $(-2, 1)$ and $(2, 1)$; no rel. max
5. Inc. on $(-\infty, -2)$ and $(1, \infty)$; dec. on $(-2, 1)$; rel max. when $x = -2$; rel. min. when $x = 1$
7. Dec. on $(-\infty, -1)$; inc. on $(-1, 3)$ and $(3, \infty)$; rel. min. when $x = -1$
9. Inc. on $(-\infty, 0)$ and $(0, \infty)$; no rel. min. or max
11. Inc. on $\left(-\infty, \frac{1}{2}\right)$; dec. on $\left(\frac{1}{2}, \infty\right)$; rel. max. when $x = \frac{1}{2}$
13. Dec. on $(-\infty, -5)$ and $(1, \infty)$; inc. on $(-5, 1)$; rel. min. when $x = -5$; rel. max. when $x = 1$
15. Dec. on $(-\infty, -1)$ and $(0, 1)$; inc. on $(-1, 0)$ and $(1, \infty)$; rel. max. when $x = 0$; rel. min. when $x = \pm 1$
17. Inc. on $(-\infty, 1)$ and $(3, \infty)$; dec. on $(1, 3)$; rel. max. when $x = 1$; rel. min. when $x = 3$
19. Inc. on $\left(-\infty, -\frac{2}{3}\right)$ and $\left(\frac{5}{2}, \infty\right)$; dec. on $\left(-\frac{2}{3}, \frac{5}{2}\right)$; rel. max. when $x = -\frac{2}{3}$; rel. min. when $x = \frac{5}{2}$
21. Inc. on $(-\infty, 5 - \sqrt{3})$ and $(5 + \sqrt{3}, \infty)$; dec. on $(5 - \sqrt{3}, 5 + \sqrt{3})$; rel. max. when $x = 5 - \sqrt{3}$; rel. min. when $x = 5 + \sqrt{3}$
23. Inc. on $(-\infty, -1)$ and $(1, \infty)$; dec. on $(-1, 0)$ and $(0, 1)$; rel. max. when $x = -1$; rel. min. when $x = 1$
25. Dec. on $(-\infty, -4)$ and $(0, \infty)$; inc. on $(-4, 0)$; rel. min. when $x = -4$; rel. max. when $x = 0$
27. Inc. on $(-\infty, -\sqrt{2})$ and $(0, \sqrt{2})$; dec. on $(-\sqrt{2}, 0)$ and $(\sqrt{2}, \infty)$; rel. max. when $x = \pm\sqrt{2}$; rel. min. when $x = 0$

47.



49.



55. $x \approx \pm 2.45$, $x \approx 0.67$, $y = 2$ **57.** $y \approx 0.48$

EXERCISE 13.6 (page 674)

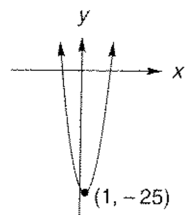
1. 41 and 41 **3.** 300 ft by 250 ft **5.** 100 units
7. \$15 **9. (a)** 110 grams; **(b)** $51\frac{9}{11}$ grams
11. 525 units; price = \$51; profit = \$10,525 **13.** \$22
15. 120 units; \$86,000 **17.** 625 units; \$4
19. \$17; \$86,700 **21.** 4 ft by 4 ft by 2 ft
23. 2 in.; 128 in^3.
27. 130 units, $p = \$340$, $P = \$36{,}980$; 125 units, $p = \$350$, $P = \$34{,}175$ **29.** 250 per lot (4 lots) **31.** 35
33. 60 mi/h **35.** 7; \$1000
37. $5 - \sqrt{3}$ tons; $5 - \sqrt{3}$ tons **41.** 10 cases; \$50.55

REVIEW PROBLEMS—CHAPTER 13 (page 680)

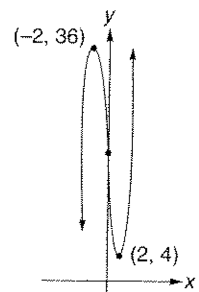
1. $y = 3$, $x = 4$, $x = -4$ **3.** $y = \frac{5}{9}$, $x = -\frac{2}{3}$

5. $x = 0$ **7.** $x = -\frac{15}{8}, -1$

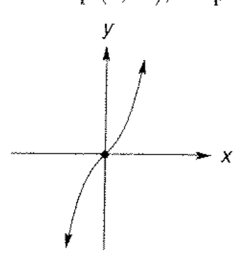
9. Inc. $(-1, 7)$; dec. on $(-\infty, -1)$ and $(7, \infty)$
11. Dec. on $(-\infty, -\sqrt{6})$, $(0, \sqrt{3})$, $(\sqrt{3}, \sqrt{6})$; inc. on $(-\sqrt{6}, -\sqrt{3})$, $(-\sqrt{3}, 0)$, $(\sqrt{6}, \infty)$
13. Conc. up on $(-\infty, 0)$ and $\left(\frac{1}{2}, \infty\right)$; conc. down on $\left(0, \frac{1}{2}\right)$
15. Conc. down on $\left(-\infty, \frac{1}{2}\right)$; conc. up on $\left(\frac{1}{2}, \infty\right)$
17. Conc. up on $\left(-\infty, -\frac{5}{4}\right)$, $\left(-\frac{1}{4}, \infty\right)$; conc. down on $\left(-\frac{5}{4}, -\frac{1}{4}\right)$
19. Rel. max. at $x = 1$; rel. min. at $x = 2$
21. Rel. min. at $x = -1$
23. Rel. max. at $x = -\frac{2}{5}$; rel. min. at $x = 0$
25. At $x = 3$ **27.** At $x = 1$ **29.** At $x = 2 \pm \sqrt{2}$
31. Maximum: $f(2) = 16$; minimum: $f(1) = -1$
33. Maximum: $f(0) = 0$; minimum: $f\left(-\frac{6}{5}\right) = -\frac{1}{120}$
35. (a) f has no relative extrema;
(b) f is conc. down on $(1, 3)$; inf. pts.: $(1, 2e^{-1})$, $(3, 10e^{-3})$
37. Int. $(-4, 0)$, $(6, 0)$, $(0, -24)$; inc. $(1, \infty)$; dec. $(-\infty, 1)$; rel. min. when $x = 1$; conc. up $(-\infty, \infty)$



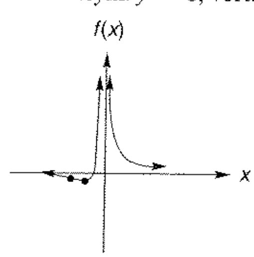
39. Int. $(0, 20)$; inc. $(-\infty, -2)$, $(2, \infty)$; dec. $(-2, 2)$; rel. max. when $x = -2$; rel. min. when $x = 2$; conc. up $(0, \infty)$; conc. down $(-\infty, 0)$; inf. pt. when $x = 0$



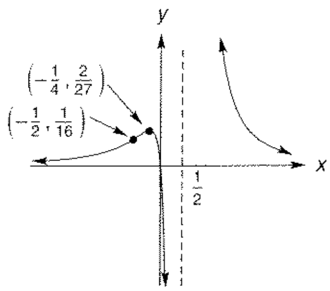
41. Int. $(0, 0)$; inc. $(-\infty, \infty)$; conc. down $(-\infty, 0)$; conc. up $(0, \infty)$; inf. pt. when $x = 0$; sym. about origin



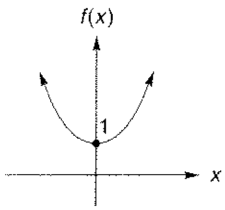
43. Int. $(-5, 0)$; inc. $(-10, 0)$; dec. $(-\infty, -10)$, $(0, \infty)$; rel. min. when $x = -10$; conc. up $(-15, 0)$, $(0, \infty)$; conc. down $(-\infty, -15)$; inf. pt. when $x = -15$; horiz. asym. $y = 0$; vert. asym. $x = 0$



45. Int. $(0, 0)$; inc. $\left(-\infty, -\frac{1}{4}\right)$; dec. $\left(-\frac{1}{4}, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$; rel. max. when $x = -\frac{1}{4}$; conc. up $\left(-\infty, -\frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$; conc. down $\left(-\frac{1}{2}, \frac{1}{2}\right)$; inf. pt. when $x = -\frac{1}{2}$; horiz. asym. $y = 0$; vert. asym. $x = \frac{1}{2}$



47. Int. $(0, 1)$; inc. $(0, \infty)$; dec. $(-\infty, 0)$; rel. min. when $x = 0$; conc. up $(-\infty, \infty)$; sym. about y-axis



49. **(a)** False; **(b)** false; **(c)** true; **(d)** false; **(e)** false
51. $q > 2$
57. Rel. max. $(-1.32, 12.28)$; rel. min. $(0.44, 1.29)$
59. $x \approx -0.60$ **61.** 20 **63.** 200 **65.** \$2800
67. 100 ft by 200 ft **69.** **(a)** 200 stands at \$120 per stand; **(b)** 300 stands

MATHEMATICAL SNAPSHOT—CHAPTER 13 (page 684)

1. The data for 1998–2000 fall into the same pattern as the 1959–1969 data

EXERCISE 14.1 (page 691)

1. $3\,dx$ **3.** $\frac{2x^3}{\sqrt{x^4 - 9}}\,dx$ **5.** $-\frac{2}{x^3}\,dx$
7. $\frac{2x}{x^2 + 7}\,dx$ **9.** $3e^{2x^3+3}(12x^2 + 4x + 3)\,dx$
11. $\Delta y = -0.14$, $dy = -0.14$
13. $\Delta y = -2.5$, $dy = -2.75$
15. $\Delta y \approx 0.049$, $dy = 0.050$ **17.** **(a)** -1; **(b)** 2.9
19. 9.95 **21.** $4\frac{1}{32}$ **23.** -0.03 **25.** 1.01 **27.** $\frac{1}{2}$
29. $\frac{1}{6p(p^2 + 5)^2}$ **31.** $-p^2$ **33.** $\frac{1}{36}$ **35.** $-\frac{4}{5}$
37. 44; 41.8 **39.** 2.04 **41.** 0.7
43. $(1.69 \times 10^{-11})\pi \text{ cm}^3$ **45.** **(c)** 42 units

APPLICATIONS IN PRACTICE 14.2

1. $\int 28.3\,dq = 28.3q + C$
2. $\int 0.12t^2\,dt = 0.04t^3 + C$
3. $\int -\frac{480}{t^3}\,dt = \frac{240}{t^2} + C$
4. $\int (500 + 300\sqrt{t})\,dt = 500t + 200t^{3/2} + C$
5. $S(t) = 0.7t^3 - 32.7t^2 + 491.6t + C$

EXERCISE 14.2 (page 698)

1. $7x + C$ **3.** $\frac{x^9}{9} + C$ **5.** $-\frac{5}{6x^6} + C$
7. $-\frac{2}{9x^9} + C$ **9.** $-\frac{5}{6y^{6/5}} + C$ **11.** $4t + \frac{t^2}{2} + C$
13. $\frac{y^6}{6} - \frac{5y^2}{2} + C$ **15.** $t^3 - 2t^2 + 5t + C$
17. $(7 + e)x + C$ **19.** $\frac{x^2}{14} - \frac{3x^5}{20} + C$
21. $6e^x + C$ **23.** $\frac{x^{9.3}}{9.3} - \frac{9x^7}{7} - \frac{1}{x^3} - \frac{1}{2x^2} + C$
25. $-\frac{4x^{3/2}}{9} + C$ **27.** $\frac{x^{3/4}}{3} + C$
29. $\frac{x^4}{12} + \frac{3}{2x^2} + C$ **31.** $\frac{w^3}{2} + \frac{2}{3w} + C$
33. $\frac{1}{7}(z^2 - 5z) + C$ **35.** $\frac{u^{e+1}}{e + 1} + e^u + C$
37. $\frac{4x^{3/2}}{3} - \frac{12x^{5/4}}{5} + C$
39. $-\frac{3x^{5/3}}{25} - 7x^{1/2} + 3x^2 + C$
41. $\frac{x^4}{4} - x^3 + \frac{5x^2}{2} - 15x + C$ **43.** $\frac{2x^{5/2}}{5} + 2x^{3/2} + C$
45. $\frac{4u^3}{3} + 2u^2 + u + C$ **47.** $\frac{2v^3}{3} + 3v + \frac{1}{2v^4} + C$
49. $\frac{z^3}{6} + \frac{5z^2}{2} + C$ **51.** $x + e^x + C$
53. No, $F(x) - G(x)$ might be a nonzero constant
55. $\frac{1}{\sqrt{x^2 + 1}} + C$

APPLICATIONS IN PRACTICE 14.3

1. $N(t) = 800t + 200e^t + 6317.37$
2. $y(t) = 14t^3 + 12t^2 + 11t + 3$

EXERCISE 14.3 (page 703)

1. $y = \frac{3x^2}{2} - 4x + 1$ **3.** 18
5. $y = -\frac{x^4}{4} + \frac{2x^3}{3} + x + \frac{19}{12}$
7. $y = \frac{x^4}{12} + x^2 - 5x + 13$ **9.** $p = 0.7$
11. $p = 275 - 0.5q - 0.1q^2$ **13.** $c = 1.35q + 200$

APPLICATIONS IN PRACTICE 14.7

1. \$5975

EXERCISE 14.7 (page 728)

1. $\frac{2}{3}$ square unit **3.** $\frac{15}{32}$ square unit

5. $S_n = \frac{1}{n}\left[4\left(\frac{1}{n}\right) + 4\left(\frac{2}{n}\right) + \cdots + 4\left(\frac{n}{n}\right)\right] = \frac{2(n+1)}{n}$

7. **(a)** $S_n = \frac{n+1}{2n} + 1$; **(b)** $\frac{3}{2}$ **9.** $\frac{1}{2}$ square unit

11. $\frac{1}{3}$ square unit **13.** $\frac{16}{3}$ square units **15.** 20

17. −18 **19.** $\frac{5}{6}$ **21.** 0 **23.** $\frac{11}{4}$

25. 4.3 square units **27.** 2.4 **29.** −25.5

APPLICATIONS IN PRACTICE 14.8

1. \$32,830 **2.** \$28,750

EXERCISE 14.8 (page 735)

1. 14 **3.** $\frac{15}{2}$ **5.** −20 **7.** $\frac{7}{3}$ **9.** $\frac{15}{2}$

11. $\frac{4}{3}$ **13.** 0 **15.** $\frac{5}{3}$ **17.** $\frac{32}{3}$ **19.** $-\frac{1}{6}$

21. $4 \ln 8$ **23.** e^5 **25.** $\frac{1}{3}(e^8 - 1)$ **27.** $\frac{3}{4}$

29. $\frac{38}{9}$ **31.** $\frac{15}{28}$ **33.** $\frac{1}{2}\ln 3$ **35.** $\frac{1}{2}\left(e + \frac{1}{e} - 2\right)$

37. $3 - \frac{2}{e} + \frac{1}{e^2}$ **39.** $\frac{e^3}{2}(e^{12} - 1)$ **41.** $6 + \ln 19$

43. $\frac{47}{12}$ **45.** $6 - 3e$ **47.** 7 **49.** 0 **51.** $\alpha^{5/2}T$

53. $\int_a^b -Ax^{-B}dx$ **55.** \$8639 **57.** 1,973,333

59. \$220 **61.** \$2000 **63.** 696; 492 **65.** $2Ri$

69. 0.05 **71.** 3.52 **73.** 55.39

APPLICATIONS IN PRACTICE 14.9

1. 76.90 feet **2.** 5.77 grams

EXERCISE 14.9 (page 743)

1. 413 **3.** 0.340; $\frac{1}{3} \approx 0.333$ **5.** $\approx$ 0.767; 0.750

7. 0.883 **9.** 2,430,733 **11.** 3.0 square units **13.** $\frac{8}{3}$

15. 0.771 **17.** $\frac{35}{6}$ km^2

EXERCISE 14.10 (page 747)

*In Problems **1–33**, answers are assumed to be expressed in square units.*

1. 8 **3.** $\frac{19}{2}$ **5.** 8 **7.** $\frac{19}{3}$ **9.** 9 **11.** $\frac{50}{3}$

13. $\frac{125}{6}$ **15.** 8 **17.** $\frac{32}{3}$ **19.** 1 **21.** 18

23. $\frac{26}{3}$ **25.** $\frac{3}{2}\sqrt[3]{2}$ **27.** $e^2 - 1$

29. $\frac{3}{2} + 2\ln 2 = \frac{3}{2} + \ln 4$ **31.** 68 **33.** 2

35. 19 square units **37.** **(a)** $\frac{1}{16}$; **(b)** $\frac{3}{4}$; **(c)** $\frac{7}{16}$

39. **(a)** $\ln\frac{5}{3}$; **(b)** $\ln 4 - 1$; **(c)** $2 - \ln 3$

41. 1.89 square units **43.** 11.41 square units

EXERCISE 14.11 (page 754)

1. Area $= \int_{-2}^{3}[(x+6) - x^2]dx$

3. Area $= \int_0^3 [2x - (x^2 - x)]dx + \int_3^4 [(x^2 - x) - 2x]dx$

5. Area $= \int_0^1 [(y+1) - \sqrt{1-y}]dy$

7. Area $= \int_1^2 [(7 - 2x^2) - (x^2 - 5)]dx$

*In Problems **9–33**, answers are assumed to be expressed in square units.*

9. $\frac{4}{3}$ **11.** $\frac{16}{3}$ **13.** $8\sqrt{6}$ **15.** 40 **17.** $\frac{125}{6}$

19. $\frac{9}{2}$ **21.** $\frac{125}{12}$ **23.** $\frac{32}{81}$ **25.** $\frac{44}{3}$

27. $\frac{4}{3}(5\sqrt{5} - 2\sqrt{2})$ **29.** $\frac{1}{2}$ **31.** $\frac{255}{32} - 4\ln 2$

33. 12 **35.** $\frac{20}{63}$ **37.** $\frac{3}{2m^3}$ square units **39.** $2^{4/3}$

41. 4.76 square units **43.** 6.17 square units

EXERCISE 14.12 (page 758)

1. CS = 25.6, PS = 38.4

3. CS = 50 ln 2 − 25, PS = 1.25

5. CS = 800, PS = 1000 **7.** \$426.67 **9.** \$254,000

11. CS $\approx$ 1197, PS $\approx$ 477

REVIEW PROBLEMS—CHAPTER 14 (page 761)

1. $\frac{x^4}{4} + x^2 - 7x + C$ **3.** $\frac{256}{3}$

5. $-3(x+5)^{-2} + C$ **7.** $2\ln|x^3 - 6x + 1| + C$

9. $\frac{11\sqrt[3]{11}}{4} - 4$ **11.** $\frac{y^4}{4} + \frac{2y^3}{3} + \frac{y^2}{2} + C$

13. $\frac{4z^{3/4}}{3} - \frac{6z^{5/6}}{5} + C$ **15.** $\frac{1}{3}\ln\frac{57}{5}$

17. $\frac{2}{27}(3x^3 + 2)^{3/2} + C$ **19.** $\frac{1}{2}(e^{2y} + e^{-2y}) + C$

21. $\ln|x| - \frac{2}{x} + C$ **23.** 111 **25.** $\frac{7}{3}$

27. $4 - 3\sqrt[3]{2}$ **29.** $\frac{3}{t} - \frac{2}{\sqrt{t}} + C$ **31.** $\frac{3}{2} - 5\ln 2$

33. $4(x^{3/2} + 1)^{3/2} + C$ **35.** 1 **37.** $\frac{(1 + e^{3x})^3}{9} + C$

39. $\frac{2\sqrt{10^{3x}}}{\ln 10} + C$ **41.** $y = \frac{1}{2}e^{2x} + 3x - 1$

In Problems 43–57, answers are assumed to be expressed in square units.

43. $\frac{4}{3}$ **45.** $\frac{16}{3}$ **47.** $\frac{125}{6}$ **49.** $6 + \ln 3$ **51.** $\frac{2}{3}$

53. 36 **55.** $\frac{125}{3}$ **57.** $e - 1$

59. $p = 100 - \sqrt{2q}$ **61.** \$1900 **63.** 0.5507

65. 15 square units **67.** $CS = 166\frac{2}{3}$, $PS = 53\frac{1}{3}$

73. 24.71 square units **75.** $CS \approx 1148$, $PS \approx 251$

MATHEMATICAL SNAPSHOT—CHAPTER 14 (page 764)

1. **(a)** 225; **(b)** 125

3. **(a)** \$2,002,500; **(b)** 18,000; **(c)** \$111.25

APPLICATIONS IN PRACTICE 15.1

1. $S(t) = -40te^{0.1t} + 400e^{0.1t} + 4600$

2. $P(t) = 0.025t^2 - 0.05t^2 \ln t + 0.05t^2(\ln t)^2 + C$

EXERCISE 15.1 (page 770)

1. $\frac{2}{3}x(x+5)^{3/2} - \frac{4}{15}(x+5)^{5/2} + C$

3. $-e^{-x}(x+1) + C$ **5.** $\frac{y^4}{4}\left[\ln(y) - \frac{1}{4}\right] + C$

7. $x[\ln(4x) - 1] + C$

9. $10x(x+1)^{3/2} - 4(x+1)^{5/2} + C$
$= 2(x+1)^{3/2}(3x-2) + C$

11. $-\frac{x}{10(5x+2)^2} - \frac{1}{50(5x+2)} + C$

13. $-\frac{1}{x}(1 + \ln x) + C$ **15.** $e^2(3e^2 - 1)$

17. $\frac{1}{2}(1 - e^{-1})$, parts not needed

19. $2(9\sqrt{3} - 10\sqrt{2})$

21. $2x(x-1)\ln(x-1) - x^2 + C$

23. $e^x(x^2 - 2x + 2) + C$

25. $\frac{x^3}{3} + 2e^{-x}(x+1) - \frac{e^{-2x}}{2} + C$

27. $\frac{e^{x^2}}{2}(x^2 - 1) + C$

29. $\frac{2^{2x-1}}{\ln 2} + \frac{2^{x+1}x}{\ln 2} - \frac{2^{x+1}}{\ln^2 2} + \frac{x^3}{3} + C$

31. $2e^3 + 1$ square units **33.** $\frac{298}{15}$ square units

37. $\int f^{-1}(x)dx = xf^{-1}(x) - F(f^{-1}(x)) + C$

APPLICATIONS IN PRACTICE 15.2

1. $r(q) = \frac{5}{2}\ln\left|\frac{3(q+1)^3}{q+3}\right|$

2. $V(t) = 150t^2 - 900\ln(t^2 + 6) + C$

EXERCISE 15.2 (page 777)

1. $\frac{12}{x+6} - \frac{2}{x+1}$ **3.** $1 + \frac{2}{x+2} - \frac{8}{x+4}$

5. $\frac{1}{x+2} + \frac{2}{(x+2)^2}$ **7.** $\frac{3}{x} - \frac{2x}{x^2+1}$

9. $2\ln|x| + 3\ln|x-1| + C = \ln|x^2(x-1)^3| + C$

11. $-3\ln|x+1| + 4\ln|x-2| + C$
$= \ln\left|\frac{(x-2)^4}{(x+1)^3}\right| + C$

13. $\frac{1}{4}\left[\frac{3x^2}{2} + 2\ln|x-1| - 2\ln|x+1|\right] + C$
$= \frac{1}{4}\left(\frac{3x^2}{2} + \ln\left[\frac{x-1}{x+1}\right]^2\right) + C$

15. $\ln|x| + 2\ln|x-4| - 3\ln|x+3| + C$
$= \ln\left|\frac{x(x-4)^2}{(x+3)^3}\right| + C$

17. $\ln|x^6 + 2x^4 - x^2 - 2| + C$, partial fractions not required

19. $\frac{4}{x-2} - 5\ln|x-1| + 7\ln|x-2| + C$
$= \frac{4}{x-2} + \ln\left|\frac{(x-2)^7}{(x-1)^5}\right| + C$

21. $4\ln|x| - \ln(x^2+4) + C = \ln\left[\frac{x^4}{x^2+4}\right] + C$

23. $-\frac{1}{2}\ln(x^2+1) - \frac{2}{x-3} + C$

25. $5\ln(x^2+1) + 2\ln(x^2+2) + C$
$= \ln[(x^2+1)^5(x^2+2)^2] + C$

27. $\frac{3}{2}\ln(x^2+1) + \frac{1}{x^2+1} + C$

29. $18\ln(4) - 10\ln(5) - 8\ln(3)$

31. $11 + 24\ln\frac{2}{3}$ square units

EXERCISE 15.3 (page 784)

1. $\frac{x}{9\sqrt{9-x^2}} + C$ **3.** $-\frac{\sqrt{16x^2+3}}{3x} + C$

5. $\frac{1}{6}\ln\left|\frac{x}{6+7x}\right| + C$ **7.** $\frac{1}{3}\ln\left|\frac{\sqrt{x^2+9}-3}{x}\right| + C$

9. $\frac{1}{2}\left[\frac{4}{5}\ln|4+5x| - \frac{2}{3}\ln|2+3x|\right] + C$

11. $\frac{1}{8}(2x - \ln[4 + 3e^{2x}]) + C$

13. $7\left[\frac{1}{5(5+2x)} + \frac{1}{25}\ln\left|\frac{x}{5+2x}\right|\right] + C$

15. $1 + \ln\frac{4}{9}$

17. $\frac{1}{2}(x\sqrt{x^2-3} - 3\ln|x + \sqrt{x^2-3}|) + C$

19. $\frac{1}{144}$ **21.** $e^x(x^2 - 2x + 2) + C$

23. $2\left(-\frac{\sqrt{4x^2+1}}{2x} + \ln|2x + \sqrt{4x^2+1}|\right) + C$

25. $\frac{1}{9}\left(\ln|1+3x| + \frac{1}{1+3x}\right) + C$

27. $\frac{1}{\sqrt{5}}\left(\frac{1}{2\sqrt{7}}\ln\left|\frac{\sqrt{7}+\sqrt{5}x}{\sqrt{7}-\sqrt{5}x}\right|\right) + C$

29. $\frac{4}{81}\left[\frac{(3x)^6 \ln(3x)}{6} - \frac{(3x)^6}{36}\right] + C$
$= x^6[6\ln(3x) - 1] + C$
31. $4(9x-2)(1+3x)^{3/2} + C$
33. $\frac{1}{2}\ln|2x + \sqrt{4x^2-13}| + C$
35. $-\frac{\sqrt{9-4x^2}}{9x} + C$
37. $\frac{1}{2\pi}(4\sqrt{x} - \ln|\pi + 7e^{4\sqrt{x}}|) + C$
39. $\frac{1}{2}\ln(x^2+1) + C$ **41.** $(2x^2+1)^{3/2} + C$
43. $\ln\left|\frac{x-3}{x-2}\right| + C$ **45.** $\frac{x^4}{4}\left[\ln(x) - \frac{1}{4}\right] + C$
47. $e^{2x}(2x-1) + C$
49. $x(\ln x)^2 - 2x\ln(x) + 2x + C$
51. $\frac{2}{3}(9\sqrt{3} - 10\sqrt{2})$ **53.** $2(2\sqrt{2} - \sqrt{7})$
55. $\frac{7}{2}\ln(2) - \frac{3}{4}$ **57.** $\ln\left|\frac{q_n(1-q_0)}{q_0(1-q_n)}\right|$
59. **(a)** \$37,599; **(b)** \$4924 **61.** **(a)** \$5481; **(b)** \$535

EXERCISE 15.4 (page 787)

1. $\frac{7}{3}$ **3.** -1 **5.** 0 **7.** 13 **9.** \$11,050
11. \$3155

APPLICATIONS IN PRACTICE 15.5

1. $I = I_0e^{-0.0085x}$

EXERCISE 15.5 (page 793)

1. $y = -\frac{1}{x^2+C}$ **3.** $y = (x^2+1)^{3/2} + C$
5. $y = Ce^x, C > 0$ **7.** $y = Cx, C > 0$
9. $y = \sqrt[3]{3x-2}$ **11.** $y = \ln\frac{x^3+3}{3}$
13. $y = \frac{4x^2+3}{2(x^2+1)}$ **15.** $y = \sqrt{\left(\frac{3x^2}{2} + \frac{3}{2}\right)^2 - 1}$
17. $y = \ln\left(\frac{1}{2}\sqrt{x^2+3}\right)$ **19.** $c = (q+1)e^{1/(q+1)}$
21. 120 weeks
23. $N = 40{,}000e^{0.018t}$; $N = 40{,}000(1.2)^{t/10}$; 57,600
25. $2e^{1.08124}$ billion **27.** 0.01204; 57.57 sec
29. 2900 years **31.** $N = N_0e^{k(t-t_0)}, t \geq t_0$
33. 12.6 units **35.** $A = 400(1 - e^{-t/2})$, 157 g/m²
37. **(a)** $V = 21{,}000e^{(2\ln 0.9)t}$; **(b)** June 2002

EXERCISE 15.6 (page 801)

1. 58,800 **3.** 500 **5.** 1990 **7.** **(b)** 375
9. 3:21 A.M **11.** \$62,500
13. $N = M - (M - N_0)e^{-kt}$

APPLICATIONS IN PRACTICE 15.7

1. 20 ml

EXERCISE 15.7 (page 805)

1. $\frac{1}{3}$ **3.** Div **5.** $\frac{1}{e}$ **7.** Div **9.** $-\frac{1}{2}$ **11.** 0
13. **(a)** 800; **(b)** $\frac{2}{3}$ **15.** 4,000,000 **17.** $\frac{1}{3}$ square unit
19. 20,000 increase

REVIEW PROBLEMS—CHAPTER 15 (page 808)

1. $\frac{x^2}{4}[2\ln(x) - 1] + C$ **3.** $5 + \frac{9}{4}\ln 3$
5. $\ln|3x+1| + 4\ln|x-2| + C$
7. $\frac{1}{2(x+2)} + \frac{1}{4}\ln\left|\frac{x}{x+2}\right| + C$
9. $-\frac{\sqrt{9-16x^2}}{9x} + C$ **11.** $\frac{3}{2}\ln\left|\frac{x-3}{x+3}\right| + C$
13. $e^{7x}(7x-1) + C$ **15.** $\frac{1}{2}\ln|\ln 2x| + C$
17. $x - \frac{3}{2}\ln|3+2x| + C$
19. $2\ln|x| + \frac{3}{2}\ln(x^2+1) + C$
21. $2\sqrt{x+1}[\ln(x+1) - 2] + C$ **23.** 34
25. $y = Ce^{x^3+x^2}, C > 0$ **27.** $\frac{1}{18}$ **29.** Div
31. 144,000 **33.** 0.0005; 90%
35. $N = \frac{450}{1 + 224e^{-1.02t}}$ **37.** 4:16 P.M **39.** 1
41. **(a)** 207, 208; **(b)** 157, 165; **(c)** 41, 41

MATHEMATICAL SNAPSHOT—CHAPTER 15 (page 810)

1. 114; 69 **5.** Answers may vary

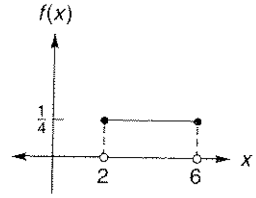
APPLICATIONS IN PRACTICE 16.1

1. $\frac{1}{3}$ **2.** 0.607
3. Mean 5 years, standard deviation 5 years

EXERCISE 16.1 (page 819)

1. **(a)** $\frac{5}{12}$; **(b)** $\frac{11}{16} = 0.6875$; **(c)** $\frac{13}{16} = 0.8125$;
(d) $-1 + \sqrt{10}$

3. **(a)** $f(x) = \begin{cases} \frac{1}{4}, & \text{if } 2 \leq x \leq 6 \\ 0, & \text{otherwise} \end{cases}$



(b) $\frac{1}{4}$; **(c)** 0; **(d)** $\frac{3}{8}$; **(e)** $\frac{3}{4}$; **(f)** 0; **(g)** 1; **(h)** 4; **(i)** $\frac{2}{\sqrt{3}}$;

EXERCISE 17.7 (page 871)

1. $\left(\frac{14}{3}, -\frac{13}{3}\right)$ **3.** $(2, 5), (2, -6), (-1, 5), (-1, -6)$

5. $(50, 150, 350)$ **7.** $\left(-2, \frac{3}{2}\right)$, rel. min.

9. $\left(-\frac{1}{4}, \frac{1}{2}\right)$, rel. max.

11. $\left(\frac{2}{5}, -\frac{3}{5}\right)$; $D = -5 < 0$ no relative extremum

13. $(0, 0)$, rel. max.; $\left(4, \frac{1}{2}\right)$, rel. min.; $\left(0, \frac{1}{2}\right)$, $(4, 0)$, neither

15. $(122, 127)$, rel. max. **17.** $(-1, -1)$, rel. min.

19. $(0, -2)$, $(0, 2)$, neither **21.** $l = 24, k = 14$

23. $p_A = 80, p_B = 85$

25. $q_A = 48, q_B = 40, p_A = 52, p_B = 44$, profit $= 3304$

27. $q_A = 3, q_B = 2$ **29.** 1 ft by 2 ft by 3 ft

31. $\left(\frac{105}{37}, \frac{28}{37}\right)$, rel. min. **33.** $a = -8, b = -12, d = 33$

35. **(a)** 2 units of A and 3 units B;
(b) Selling price for A is 30 and selling price for B is 19. Relative maximum profit is 25.

37. **(a)** $P = 5T(1 - e^{-x}) - 20x - 0.1T^2$;
(c) Relative maximum at $(20, \ln 5)$; no relative extremum at $\left(5, \ln\frac{5}{4}\right)$

EXERCISE 17.8 (page 879)

1. $(2, -2)$ **3.** $\left(3, \frac{3}{2}, -\frac{3}{2}\right)$ **5.** $\left(0, \frac{1}{4}, \frac{5}{8}\right)$

7. $(6, 3, 2)$ **9.** $\left(\frac{2}{3}, \frac{4}{3}, -\frac{4}{3}\right)$ **11.** $(3, 3, 6)$

13. Plant 1, 40 units; plant 2, 60 units

15. 74 units (when $l = 8, k = 7$)

17. \$15,000 on newspaper advertising and \$45,000 on TV advertising

19. $x = 5, y = 15, z = 5$

21. $x = 12, y = 8$ **23.** $x = 10, y = 20, z = 5$

EXERCISE 17.9 (page 887)

1. $\hat{y} = 0.98 + 0.61x$; 3.12 **3.** $\hat{y} = 0.057 + 1.67x$; 5.90

5. $\hat{q} = 82.6 - 0.641p$ **7.** $\hat{y} = 100 + 0.13x$; 105.2

9. $\hat{y} = 8.5 + 2.5x$

11. **(a)** $\hat{y} = 35.9 - 2.5x$; **(b)** $\hat{y} = 28.4 - 2.5x$

EXERCISE 17.11 (page 893)

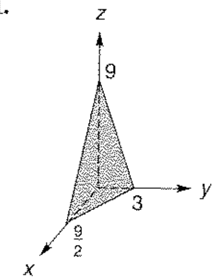
1. 18 **3.** $\frac{1}{4}$ **5.** $\frac{2}{3}$ **7.** 3 **9.** $\frac{525}{2}$ **11.** $-\frac{58}{5}$

13. $\frac{8}{3}$ **15.** -1 **17.** $\frac{e^2}{2} - e + \frac{1}{2}$ **19.** $-\frac{27}{4}$

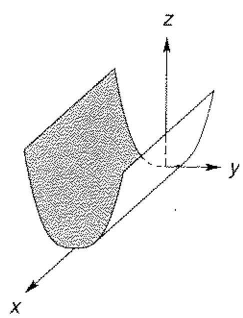
21. $\frac{1}{24}$ **23.** $e^{-4} - e^{-2} - e^{-3} + e^{-1}$ **25.** $\frac{3}{8}$

REVIEW PROBLEMS—CHAPTER 17 (page 895)

1.



3.



5. $8x + 6y$; $6x + 2y$ **7.** $\frac{y}{(x+y)^2}$; $-\frac{x}{(x+y)^2}$

9. $\frac{y}{x^2 + y^2}$ **11.** $2xze^{x^2yz}(1 + x^2yz)$ **13.** $2(x + y)$

15. $e^{x+y+z}\left[\ln xyz + \frac{1}{y}\right]$; $e^{x+y+z}\left[\ln xyz + \frac{1}{x} + \frac{1}{z}\right]$

17. $\frac{1}{64}$ **19.** $2(x + y)e^r + 2\left(\frac{x + 3y}{r + s}\right)$; $2\left(\frac{x + 3y}{r + s}\right)$

21. $\frac{2x + 2y + z}{4z - x}$ **23.** $\frac{\partial P}{\partial l} = 14l^{-0.3}k^{0.3}$; $\frac{\partial P}{\partial k} = 6l^{0.7}k^{-0.7}$

25. Competitive **27.** $(2, 2)$, rel. min.

29. 4 ft by 4 ft by 2 ft

31. A, 89 cents per pound; B, 94 cents per pound

33. $(3, 2, 1)$ **35.** $\hat{y} = 12.67 + 3.29x$

37. 8 **39.** $\frac{1}{30}$

MATHEMATICAL SNAPSHOT—CHAPTER 17 (page 898)

1. $y = 9.50e^{-0.22399x} + 5$ **3.** $T = 79e^{-0.01113t} + 45$

EXERCISE A.1 (page 905)

1.