

4.4 Logarithmic Functions

Defn of Natural Logarithmic Function

$$\ln x = b \Leftrightarrow e^b = x$$

Properties of logarithms

① $\ln 1 = 0$

② $\ln e = 1$

③ $\ln e^x = x$

④ $e^{\ln x} = x$

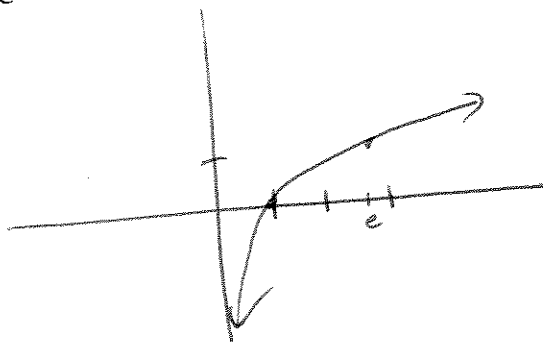
⑤ $\ln(xy) = \ln x + \ln y$

⑥ $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

⑦ $\ln x^n = n \ln x$

Ex 1 Sketch graph of $f(x)$.

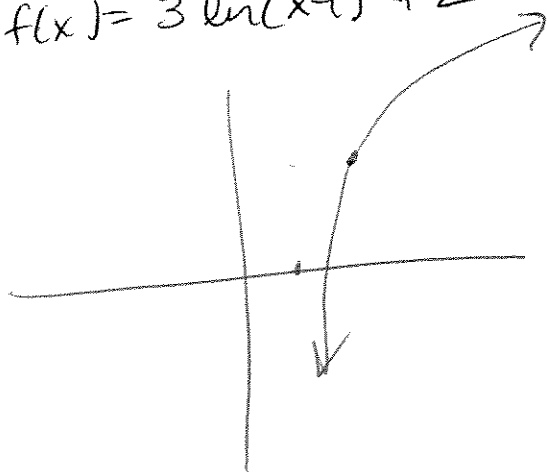
(a) $f(x) = \ln x$



$$y = \ln x \mid y = x \\ \Leftrightarrow e^y = x$$

x	y
1	0
e	1
$\frac{1}{e}$	-1

(b) $f(x) = 3 \ln(x-1) + 2$



shift up 2
shift right 1
stretch by 3

4.4 (cont)

Ex 2 Simplify.

$$(a) \ln e^{3x+2} = 3x+2$$

$$(b) -8 + e^{\ln x^3} = -8 + x^3$$

Ex 3 Given $\ln 2 \approx 0.6931$ + $\ln 3 \approx 1.0986$, simplify these expressions.

$$\begin{aligned}(a) \ln 24 &= \ln(8 \cdot 3) = \ln 8 + \ln 3 \\ &= \ln 2^3 + \ln 3 \\ &= 3 \ln 2 + \ln 3 \\ &= 3(0.6931) + 1.0986 \\ &= 3.1779\end{aligned}$$

$$(b) \ln \sqrt[3]{12}$$

$$= \ln(12)^{1/3} = \frac{1}{3} \ln 12$$

$$= \frac{1}{3} \ln(4 \cdot 3) = \frac{1}{3} [\ln 4 + \ln 3]$$

$$= \frac{1}{3} [\ln 2^2 + \ln 3]$$

$$= \frac{1}{3} [2 \ln 2 + \ln 3] = \frac{1}{3} [2(0.6931) + 1.0986] \approx 0.8283$$

4.4 (cont)

Ex 4 Use log properties to rewrite these expressions.

$$(a) \ln \sqrt{\frac{x^3}{x+1}} = \ln \left(\frac{x^3}{x+1} \right)^{1/2} = \frac{1}{2} \ln \left(\frac{x^3}{x+1} \right)$$

$$= \frac{1}{2} [\ln(x^3) - \ln(x+1)]$$

$$= \frac{1}{2} [3 \ln x - \ln(x+1)]$$

$$(b) \ln \left(\frac{xy}{z} \right) = \ln(xy) - \ln(z) = \ln x + \ln y - \ln z$$

$$(c) \ln \left(\frac{2x}{\sqrt{x^2-1}} \right) = \ln(2x) - \ln(x^2-1)^{1/2}$$
$$= \ln(2x) - \frac{1}{2} \ln(x^2-1)$$

$$(d) \ln \left(\frac{1}{3} \right) = \ln 1 - \ln 3 = 0 - \ln 3 = -\ln 3$$

4.4 (cont)

Ex 5 Use log properties to rewrite these expressions.

$$(a) [\ln(x+3) - \ln(x+4) + \ln x]^2$$

$$= \left(\ln \left(\frac{x+3}{x+4} \right) + \ln x \right)^2$$

$$= \left(\ln \left[\frac{x(x+3)}{x+4} \right] \right)^2 = 2 \ln \left[\frac{x(x+3)}{x+4} \right]$$
$$= \ln \left[\frac{x(x+3)}{x+4} \right]^2$$

$$(b) 2 \ln x + 3 \ln y - 5 \ln z$$

$$= \ln x^2 + \ln y^3 - \ln z^5$$

$$= \ln(x^2 y^3) - \ln z^5 = \ln \left(\frac{x^2 y^3}{z^5} \right)$$

$$(c) \frac{1}{2} \ln(x-2) + \frac{3}{2} \ln(x+2)$$

$$= \ln(x-2)^{1/2} + \ln(x+2)^{3/2}$$

$$= \ln \left[(x-2)^{1/2} (x+2)^{3/2} \right]$$

4.4 (cont)

Ex 6 Solve these equations.

(a) $e^{\ln x^2} - 9 = 0$

$$e^{\ln x^2} = 9$$

$$x^2 = 9$$

$$x = \pm 3$$

★ isolate exponential first

(b) $2e^{-x+1} - 5 = 9$

$$\frac{2e^{-x+1}}{2} = \frac{14}{2}$$

$$e^{-x+1} = 7$$

$$\ln(e^{-x+1}) = \ln 7$$

$$-x+1 = \ln 7$$

$$-x = \ln 7 - 1$$

$$x = -(\ln 7 - 1)$$

$$x = 1 - \ln 7$$

(c) $\frac{50}{1+12e^{-0.02x}} = 10.5$

$$\frac{50}{10.5} = \frac{10.5(1+12e^{-0.02x})}{10.5}$$

$$4.7619 = 1+12e^{-0.02x}$$

$$\frac{3.7619}{12} = \frac{12e^{-0.02x}}{12}$$

$$0.3135 = e^{-0.02x}$$

$$\ln 0.3135 = \frac{-0.02x}{-0.02}$$

$$57.999 \approx x$$

Math/100
100