6.1 Intro to Discrete Probability

* this section assumes we have finitely many outcomes that are all equally likely.

Vocab <u>experiment</u>: procedure that yields one of several outcomes

<u>sample space</u>: set of all possible outcomes (S)

<u>event</u>: subset of sample space

$\boxed{\text{Defn 1}}$

Defn 1 If is finite space of equally likely outcomes, E is an event, $E \subseteq S$, then <u>probability of E</u> is

$$p(E) = \frac{|E|}{|S|}.$$

<u>sampling w/ replacement</u>: doing an experiment where you draw the object, note its characteristic, and then put it back to be possibly "sampled" again.

<u>sampling w/o replacement</u>: doing an experiment where you don't put the object back after choosing it, thereby decreasing the # of objects w/ every draw.

Thm 1 $E \subseteq S$. $p(\overline{E}) = 1 - p(E)$

<u>pf</u> note: $|\overline{E}| = |S| - |E|$

$$p(\overline{E}) = \frac{|\overline{E}|}{|S|} = \frac{|S| - |E|}{|S|} = 1 - p(E) \,/\!/$$

6.1 (cont)

Thm 2 $E_1 \subseteq S$, $E_2 \subseteq S$.

$$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$$

Pf note: $|E_1 \cup E_2| = |E_1| + |E_2| - |E_1 \cap E_2|$

$\Rightarrow$ $p(E_1 \cup E_2) =$

Ex 1 what is probability that a card selected from a fair deck is a spade? a king? a king of spades?

6.1 (cont)

Ex2 What is probability that a 5-card poker hand contains the ace of hearts?

Ex3 What is the probability that a 5-card poker hand contains the 2 of diamonds and the 3 of spades?

Ex4 What is the probability that a die never comes up even when it is rolled 6 times?
(sampling w/ or w/o replacement?)

6.1 (cont)

Ex 5 Find the probability of winning the lottery by selecting the correct 6 integers (order does not matter) from $\{1, 2, 3, \ldots, 64\}$.

Ex 6 Which is more likely: rolling a total of 8 when 2 dice are rolled or rolling a total of 8 when 3 dice are rolled?

6.2 Probability Theory

Two conditions must be satisfied for a probability distribution function.

Let S = sample space, w/ finite # outcomes.

Assume s ("little s") is one of the outcomes in S.

① $p(s) \in [0,1] \quad \forall\ s \in S$.

② $\sum_{s \in S} p(s) = 1$

i.e. the probability of any given outcome is between 0 and 1 (or between 0% and 100%) and that all those probabilities added together is 1 (or 100%). So, $p(S) = 1$.

(note: when S is not a countable set, then we have to expand these defns to use calculus.)

Ex 1 Find probability of each outcome in S if we're tossing a loaded die, such that a 4 is 3 times as likely to appear than any of the other 5 numbers.

6.2 (cont)

<u>Defn 1</u> <u>uniform distribution</u> $|S| = n$, $p(s) = \frac{1}{n}$ $\forall s \in S$.

<u>Defn 2</u> $E \subseteq S$. $p(E) = \sum_{s \in E} p(s)$

note: if $|E| = \infty$, then $p(E) < \infty \Rightarrow \sum_{s \in E} p(s)$ is convergent series

<u>Ex2</u> Using the scenario in Ex 1, what is probability of rolling an even die?

<u>Thm 1</u> If $E_1, E_2, \ldots$ sequence of pairwise disjoint events in sample space S, then $p\left(\bigcup_i E_i\right) = \sum_i p(E_i)$

<u>Ex3</u> what is probability of these events when we randomly select a permutation of $\{1, 2, 3, 4\}$? (assume each # is equally likely)

(a) 1 precedes 4

(b) 1 precedes 4

6.2 (cont)

Ex 3 (cont)

(c) 4 precedes 1 and 4 precedes 2

(d) 4 precedes 3 and 2 precedes 1

(e) 4 precedes 1, 4 precedes 2, and 4 precedes 3