

Section B.1: Linear FunctionsLinear model and linear function

If the graph of a model is a straight line, the model is called *linear* and the function is called *linear function*. A linear function has a constant rate of change and a straight line graph. For all linear functions the rate of change is equal to the slope of the graph which is defined as the amount that the graph rises vertically for a given distance that it runs horizontally:

$$\text{rate of change} = \text{slope} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

The greater the rate of change, the steeper the slope.

Ex.1

You hike a 3-mile trail, starting at an elevation of 8000 feet. Along the way, the trail gains elevation at a rate of 650 feet per mile. What is the domain for the elevation function? From the given data, draw a graph of a linear function that could represent a model for your elevation as you hike along the trail. Does this model seem realistic?

This model assumes that elevation increases at a constant rate along the entire 3-mile trail. While an elevation change of 650 feet per mile seems reasonable as an average, the actual rate of change probably varies from point to point along the trail. Thus, the model's predictions are likely to be reasonable estimates, rather than exact values, of your elevation at different points along the trail.

Ex.2

A small store sells fresh pineapples. Based on data for pineapple prices between \$2 and \$5, the storeowners created a model in which a linear function is used to describe how the demand (number of pineapples sold per day) varies with the price. The points (\$2, 80 pineapples) and (\$5, 50 pineapples) are in the graph. Draw the graph. What is the rate of change for this function? Discuss the validity of this model.

This model seems reasonable within the domain for which the storeowners gathered data. Outside this domain, the model's predictions probably are not true. For example, the model predicts that the store could sell one pineapple per day at a price of \$9.90, but could never sell a pineapple at a price of \$10. On the other hand, the model predicts that the store could sell only 100 pineapples if they were free. This model is useful only in a limited domain.

The Change in the Dependent Variable

Change in dependent variable

The rate of change rule allows us to calculate the change in the dependent variable from the change in the independent variable:

$$\text{change in dependent variable} = \text{rate of change} \times \text{change in independent variable}$$

Ex.3

Using the linear demand in Example 2, predict the change for pineapples if the price increases by \$3.

Section B.2: General Equation for a Linear Function

Equation for a linear function

The general equation for a linear function is

$$\text{change in dependent variable} = \text{initial value} + \text{rate of change} \times \text{independent variable}$$

Ex.4

Suppose your job is to oversee an automated assembly line that manufactures computer chips. You arrive at work one day to find a stock of 25 chips that were produced during the night. If chips are produced at a constant rate of 4 chips per hour, how large is the stock of chips at any particular time during your shift?

The Algebraic Equation of a Line

Algebraic equation of a line

In algebra, x is used for the independent variable and y for the dependent variable. For a straight line, the slope is denoted by m and the initial value, or y -intercept, is denoted by b . The equation of a linear function is

$$y = mx + b$$

The equation of a general line is

$$Ax + By + C = 0$$

If $B = 0$, then the line is

$$x = -\frac{C}{A}$$

that is, $x = b$ (for example, $x = 3$).

If $B \neq 0$, then the line can be written as

$$y = -\frac{A}{B}x - \frac{C}{B}$$

that is, $y = mx + b$.

Ex.5

Draw the lines

- (1) $x = 4$
- (2) $y = 2x + 1$
- (3) $y = 3x$
- (4) $y = -x - 1$
- (5) $y = 2$

Ex.6 Alcohol Metabolism.

Alcohol is metabolized by the body in such a way that the blood alcohol content (BAC) decreases linearly. A study by the National Institute on Alcoholic Abuse and Alcoholism showed that, for a group of fasting males who consumed four drinks rapidly, the blood content rose to a maximum of $\frac{0.8g}{100mL}$ about an hour after the drinks were consumed. Three hours later, the blood alcohol content had decreased to $\frac{0.04g}{100mL}$. Find a linear model that describes the elimination of alcohol after the peak blood alcohol content is reached. According to the model, what is the blood alcohol content five hours after the peak is reached?

Ex.7

Write an equation for the linear demand function in Example 2. Then determine the price that should result in a demand of 80 pineapples per day.

Linear Functions from Two Data Points

How to find a linear function from two data points

Suppose we have two data points and we want to find a linear function that fits them. We can find the equation for this linear function by using the two data points to determine the rate of change and the initial value for the function in 3 steps:

Step 1: Let x be the independent variable and y be the dependent variable. Find the change in each variable between the two given points, and use these changes to calculate the slope (or rate of change):

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

Step 2: Substitute this slope and the numerical values of x and y from either data point into the equation $y = mx + b$. You can then solve for the y -intercept b because it will be the only unknown in the equation.

Step 3: Now use the slope and y -intercept to write the equation of the linear function in the form $y = mx + b$.

Ex.8

Until about 1850, humans used so little crude oil that we can call the amount zero, at least in comparison to the amount used since that time. By 1960, humans had used a total of 600 billion cubic meters of oil. Create a linear model that describes world oil use since 1850. Discuss the validity of the model.