Section D.1: Two Types of Arguments: Inductive and Deductive

Definition of an argument
An *argument* is a reasoned process that uses a set of facts or assumptions, called *premises*, to support a *conclusion*.

There are two types of arguments: inductive and deductive.

Definition of an inductive argument
An *inductive argument* makes a case for a general conclusion from more specific premises.

Ex.1 Inductive argument.
Premise: Birds fly up into the air but eventually come back down.
Premise: People who jump into the air fall back down.
Premise: Rocks thrown into the air come back down.
Premise: Balls thrown into the air come back down.
Conclusion: What goes up must come down.

Definition of a deductive argument
A *deductive argument* makes a case for a specific conclusion from more general premises.

Ex.2 Deductive argument.
Premise: All politicians are married.
Premise: Senator Harris is a politician.
Conclusion: Senator Harris is married.

Evaluating Inductive Argument

How to evaluate an inductive argument
An inductive argument can be analyzed only in terms of its strength. An argument is *strong* if it makes a compelling case for its conclusion. It is *weak* if its conclusion is not well supported by its premises.
An inductive argument cannot prove that its conclusion is true. At best, a strong inductive argument shows that its conclusion is *probably* true.

Ex.3 Strong.
Premise: Birds fly up into the air but eventually come back down.
Premise: People who jump into the air fall back down.
Premise: Rocks thrown into the air come back down.
Premise: Balls thrown into the air come back down.
Conclusion: What goes up must come down.

Ex.4 Not strong.
Premise: The movie director hired big stars for the lead roles in the next movie.
Premise: The movie director has a great advertising campaign planned for the next movie.
Premise: The next movie is a sequel to her last hit movie.
Conclusion: The next movie will be a hit.

Ex.5 Strong.
Premise: For thousands of years past, the San Andreas Fault has suffered a major earthquake at least once every hundred years.
Conclusion: We should expect another earthquake on the fault during the next one hundred years.

Evaluating Deductive Argument

How evaluate a deductive argument

A deductive argument can be analyzed in terms of its validity and soundness. An argument is *valid* if its conclusion follows necessarily from its premises. It is *sound* if it is valid and its premises are true.
Validity is concerned only with the logical structure of the argument: validity involves no personal judgment and has nothing to do with the truth of the premises or conclusion. Thus a deductive argument can be valid even if its conclusion is false.
A sound deductive argument provides definite proof that its conclusion is true. However, evaluating soundness often involves personal judgment.

Ex.6 Valid but not sound.
Premise: All politicians are married.
Premise: Senator Harris is a politician.
Conclusion: Senator Harris is married.

Section D.2: Tests of Validity

Tests of validity

We can test validity by examining the structure of a deductive argument in a systematic way using Venn diagrams:

- Draw a Venn diagram that represents all the information contained in the premises.
- Check to see whether the Venn diagram also contains the conclusion. If it does, then the argument is VALID. If it doesn't, then the argument is NOT VALID.

Ex.7 Valid.

Premise: All politicians are married.

Premise: Senator Harris is a politician.

Conclusion: Senator Harris is married.

Ex.8 Not valid.
Premise: All fish live in the water.
Premise: Whales are not fish.
Conclusion: Whales do not live in the water.

Ex.9 Not valid.
Premise: All 20th-century US presidents were men.
Premise: John Kennedy was a man.
Conclusion: John Kennedy was a 20th-century US president.

Conditional Deductive Argument

Definition of a conditional deductive argument

A *conditional deductive argument* is a deductive argument in which the first premise is a conditional statement "if p, then q".

Types of conditional deductive arguments
There are four basic types of conditional deductive arguments:
(a) **Affirming the Hypothesis**

<u>Premise</u>: If p, then q.
<u>Premise</u>: p is true.
<u>Conclusion</u>: q is true.

(b) **Affirming the Conclusion**

<u>Premise</u>: If p, then q.
<u>Premise</u>: q is true.
<u>Conclusion</u>: p is true.

(c) **Denying the Hypothesis**

<u>Premise</u>: If p, then q.
<u>Premise</u>: p is not true.
<u>Conclusion</u>: q is not true.

(d) **Denying the Conclusion**

<u>Premise</u>: If p, then q.
<u>Premise</u>: q is not true.
<u>Conclusion</u>: p is not true.

Ex.10 Affirming the hypothesis.
Premise: If a person lives in Chicago, then the person likes windy days.
Premise: Carlos lives in Chicago.
Conclusion: Carlos likes windy days.

Ex.11 Affirming the conclusion.
Premise: If an employ is regularly late, then he will be fired.
Premise: Robert was fired.
Conclusion: Robert was regularly late.

Ex.12 Denying the hypothesis.
Premise: If you liked the book, then you will love the movie.
Premise: You didn't like the book.
Conclusion: You will not love the movie.

Ex.13 Denying the conclusion.
<u>Premise</u>: A narcotic is habit-forming.
<u>Premise</u>: Heroin is not habit-forming.
<u>Conclusion</u>: Heroin is not a narcotic.

Deductive Argument with a Chain of Conditionals

Definition of a chain of conditionals

A deductive argument with a *chain of conditionals* is a deductive argument with the premises given by conditionals.

- VALID chain of conditionals:
  Premise: If p, then q.
  Premise: If q, then r.
  Conclusion: If p, then r.

Ex.14 Valid chain of conditionals.
Premise: If Maria Lopez is elected to the school board, then the school district will raise academic standards.
Premise: If the school district raises academic standards, then my children will benefit.
Conclusion: If Maria Lopez is elected to the school board, then my children will benefit.

- INVALID chain of conditionals:
  Premise: If p, then q.
  Premise: If r, then q.
  Conclusion: If p, then r.

Ex.15 Invalid chain of conditionals.
Premise: If you shop, then I make dinner.
Premise: If you take out the trash, then I make dinner.
Conclusion: If you shop, then you take out the trash.

Section D.3: Induction and Deduction in Mathematics

> Definition of a mathematical proof
>
> A *mathematical proof* is a deductive argument that demonstrates the truth of a certain claim, called theorem.

Ex.16 Pythagorean Theorem.

In a right triangle, the square of the longest side (called *hypothenuse*) is the sum of the squares of the other two sides.

We can use inductive arguments not to prove a theorem, but to test if a rule is true or not.

Ex.17 Test 1.
Is it true that for all numbers a and b

$$a \times b = b \times a?$$

Ex.18 Test 2.
Is it true that for any number a

$$\frac{2}{3} = \frac{2+a}{3+a}?$$