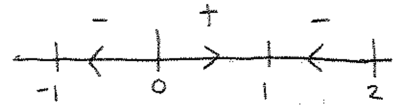
Laurel Hales
Sept 3, 2010
Homework 2
Math 5440

§1.2: 1-5

1.a $f(3/2) = f(2-1/2) = -f(1/2)$



$f(1/2) = -e^{1/2}\sin(\frac{\pi}{2}) \Rightarrow$

$\boxed{f(3/2) = -e^{1/2}}$

b. $f(-\frac{21}{2}) = -f(\frac{21}{2}) = -f(\frac{1}{2} + 2(5)) = -f(1/2)$

$-f(1/2) = -e^{1/2} \Rightarrow \boxed{f(-\frac{21}{2}) = -e^{1/2}}$

c. $f(14) = f(0+2(7)) = f(0)$

$f(0) = e^{0}\sin(0)$

$\boxed{f(14)=0}$

2. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ for any c.

$\Rightarrow \int_x^{x+2l} f(x)\,dx = \int_x^{l} f(\bar{x})\,d\bar{x} + \int_l^{x+2l} f(\bar{x})\,d\bar{x}$ $\quad y = \bar{x} - 2l$

$= \int_x^{l} f(\bar{x})\,d\bar{x} + \int_{-l}^{x} f(y-2l)\,dy$

$= \int_x^{l} f(\bar{x})\,d\bar{x} + \int_{-l}^{x} f(y)\,dy$

$\int_x^{x+2l} f(x)\,dx = \int_{-l}^{l} f(\bar{x})\,d\bar{x} = 0$ because of 2.14

3. See Back of page

3. $\begin{cases} u_{tt} - c^2 u_{xx} = 0 \\ u(x,0) = f(x) = \sin(\pi x) \\ u_t(x,0) = g(x) = 0 \\ u(0,t) = u(1,t) = 0 \end{cases}$

$u(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)] + \cancel{\frac{1}{2c}\int_{x-ct}^{x+ct} g(\bar{x})\,d\bar{x}} \to 0$

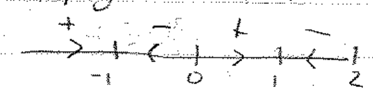
$u(x,t) = \frac{1}{2}[\sin(\pi x + \pi c t) + \sin(\pi x - \pi c t)]$

$= \frac{1}{2}[\sin(\pi x)\cos(\pi c t) + \cancel{\cos(\pi x)\sin(\pi c t)} + \sin(\pi x)\cos(\pi c t) - \cancel{\cos(\pi x)\sin(\pi c t)}]$

$= \sin(\pi x)\cos(\pi c t)$ ✓

$\sin(\pi x)$ has a period of 2 and odd reflections at 0 and 1, so it works for all x and t.

4.a. Find $u(1/2, 3/2)$ $l = 1$ $c = 1$ $f = 0$, $g(x) = x(1-x)$



$u(1/2, 3/2) = \frac{1}{2}\int_{1/2-3/2}^{1/2+3/2} g(\bar{x})\,d\bar{x} = \int_{-1}^{2} g(\bar{x})\,d\bar{x}$

$= \frac{1}{2}\int_{-1}^{0} -\bar{x}(1+\bar{x})\,d\bar{x} + \frac{1}{2}\int_0^1 \bar{x}(1-\bar{x})\,d\bar{x} + \frac{1}{2}\int_1^2 (x-2)(x-1)\,d\bar{x}$

$-g(-\bar{x})$ $\quad g(\bar{x})$ $\quad -g(2-x)$

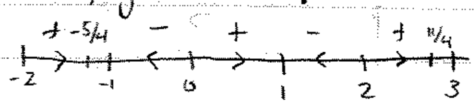
because g is odd & periodic a lot of the integral cancels

$\int_0^2 g(\bar{x})\,d\bar{x} = 0$ so,

$u(1/2, 3/2) = -\frac{1}{2}\int_{-1}^{0} x(1+x) = -\frac{1}{2}\int_{-1}^{0} x + x^2 = -\frac{1}{2}\left[\frac{x^2}{2} + \frac{x^3}{3}\right]_{-1}^{0}$

$= -\frac{1}{2}\left[\frac{1}{2} + -\frac{1}{3}\right] = -\frac{1}{2}\left[\frac{3-2}{6}\right] = \boxed{-\frac{1}{12}}$ ✓

b. Find $u(3/4, 2)$ when $l = c = 1$ $f(x) = x(1-x)$, $g(x) = x^2(1-x)$



$u(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\bar{x})\,d\bar{x}$

$u(3/4, 2) = \frac{1}{2}[f(11/4) + f(-5/4)] + \frac{1}{2}\int_{-5/4}^{11/4} g(\bar{x})\,d\bar{x}$

$= \frac{1}{2}[f(3/4 + 2) + f(3/4 - 2)] + \frac{1}{2}\int_{-5/4}^{11/4} g(\bar{x})\,d\bar{x}$

$= f(3/4) + \cancel{\frac{1}{2}\int_{-5/4}^{-5/4+2} g(\bar{x})\,d\bar{x}} \to 0$

$= f(3/4) = \frac{3}{4}(1 - 3/4)$ $\quad u(3/4, 2) = \frac{3}{16}$ ✓

5.a. u is odd and periodic with respect to x if both of the following expressions are true:

$u(x+2\ell, t) = u(x,t)$ We know that f and g are
$u(-x,t) = -u(x,t)$ both odd and have a period of 2ℓ.

$$u(x,t) = \frac{1}{2}\left[f(x+ct)+f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\bar{x})\,d\bar{x}$$
$$\stackrel{?}{=} \frac{1}{2}\left[f(x+2\ell+ct)+f(x+2\ell-ct)\right] + \frac{1}{2c}\int_{x+2\ell-ct}^{x+2\ell+ct} g(\bar{x})\,d\bar{x}$$

By eq 2.15 & 2.14 $= \frac{1}{2}\left[f(x+ct)+f(x-ct)\right] + \frac{1}{2c}\int_{x+2\ell-ct}^{x+2\ell+ct} g(\bar{x})\,d\bar{x}$ $\qquad \alpha = x+2\ell$

$$= \frac{1}{2}\left[f(x+ct)+f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\alpha-2\ell)\,d\alpha$$

$= \frac{1}{2}\left[f(x+ct)+f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\alpha)\,d\alpha$ By eq. 2.14 & 2.15.

$$u(-x,t) = \frac{1}{2}\left[f(-x+ct)+f(-x-ct)\right] + \frac{1}{2c}\int_{-x-ct}^{-x+ct} g(\bar{x})\,d\bar{x}$$
$$= \frac{1}{2}\left[-f(x-ct)-f(x+ct)\right] + \frac{1}{2c}\int_{-x-ct}^{-x+ct} g(\bar{x})\,d\bar{x}$$

$= -\frac{1}{2}\left[f(x-ct)+f(x+ct)\right] + \frac{1}{2c}\int_{-x-ct}^{-x+ct} g(\bar{x})\,d\bar{x}$ $\qquad \beta = -\bar{x}$

✓ $= -\frac{1}{2}\left[f(x-ct)+f(x+ct)\right] - \frac{1}{2c}\int_{x+ct}^{x-ct} g(-\beta)\,d\beta$

$$= -\frac{1}{2}\left[f(x-ct)+f(x+ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} -g(\beta)\,d\beta$$
$$= -\frac{1}{2}\left[f(x-ct)+f(x+ct)\right] - \frac{1}{2c}\int_{x-ct}^{x+ct} g(\beta)\,d\beta$$
$$= -u(x,t)$$

b. Show $u(x, t+\frac{2\ell}{c}) = u(x,t)$

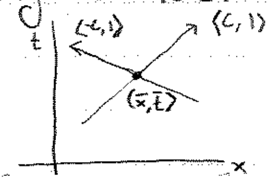
$$u(x, t+\tfrac{2\ell}{c}) = \frac{1}{2}\left[f(x+ct+2\ell)+f(x-ct-2\ell)\right] + \frac{1}{2c}\int_{x-ct-2\ell}^{x+ct+2\ell} g(\bar{x})\,d\bar{x}$$
$$= \frac{1}{2}\left[f(x+ct)+f(x-ct)\right] + \frac{1}{2c}\left[\cancelto{0}{\int_{x-ct-2\ell}^{x-ct} g(\bar{x})\,dx} + \int_{x-ct}^{x+ct} g(\bar{x})\,dx + \cancelto{0}{\int_{x+ct}^{x+ct+2\ell} g(\bar{x})\,dx}\right]$$
$$= \frac{1}{2}\left[f(x+ct)+f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\bar{x})\,d\bar{x}$$

✓ $= u(x,t)$

(3)

§1.3:1

1. Show that if u_x has a discontinuity at point $(\bar{x},\bar{t})$, then this discontinuity is propagated along at least one of the characteristics through $(\bar{x},\bar{t})$



We know that u_x is discontinuous at $(\bar{x},\bar{t})$ we also know that
$u(x,t)=\frac{1}{2}[f(x+ct)+f(x-ct)]+\frac{1}{2c}\int_{x-ct}^{x+ct} g(\bar{x})\,d\bar{x}$ from this we find:
$u_x(x,t)=\frac{1}{2}[f'(x+ct)+f'(x-ct)]+\frac{1}{2c}[g(x+ct)-g(x-ct)]$ we can split u_x into 2 differt waves, one traveling left and one traveling right:
$u_x(\bar{x},\bar{t})=[\frac{1}{2}f'(\bar{x}+c\bar{t})+\frac{1}{2c}g(\bar{x}+c\bar{t})]+[\frac{1}{2}f'(\bar{x}-c\bar{t})-\frac{1}{2c}g(\bar{x}-c\bar{t})]$ In order for $u(\bar{x},\bar{t})$ one of these 2 parts must also have a discontinuity.

If we find u_x at some point further along the right characteristic we find:

$$u_x(\bar{x}+ac,\bar{t}+a)=\frac{1}{2}[f'(\bar{x}+ac+c\bar{t}+ca)+\frac{1}{c}g(\bar{x}+ac+c\bar{t}+ca)]+\frac{1}{2}[f'(\bar{x}+ac-c\bar{t}-ca)-\frac{1}{c}g(\bar{x}+ac-c\bar{t}-ca)]$$
$$=\frac{1}{2}[f'(\bar{x}+c\bar{t}+2ac)+\frac{1}{c}g(\bar{x}+c\bar{t}+2ac)]+\frac{1}{2}[f'(\bar{x}-c\bar{t})-\frac{1}{c}g(\bar{x}-c\bar{t})]$$

So a discontinuity in the right traveling part of u_x will be propagated through the right characteristic.
It is similarly shown that a discontinuity in the left traveling part of u_x is propagated through the left characteristic.

$$u_x(\bar{x}-ac,\bar{t}+a)=\frac{1}{2}[f'(\bar{x}+c\bar{t})+\frac{1}{c}g(\bar{x}+c\bar{t})]+\frac{1}{2}[f'(x-c\bar{t}-2ac)-\frac{1}{c}g(\bar{x}-ct-2ac)]$$

✓

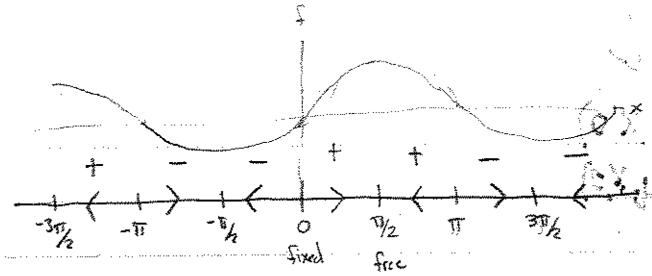
the argument ~~is~~ looks shorter if you rewrite $u(x,t)$ equivalently as

$$u(x,t)=p(x+ct)+q(x-ct) \qquad (2.3)$$

so u_x has discont. @ $(\bar{x},\bar{t}) \Rightarrow$ either $p'(\bar{x}+c\bar{t})$ or $q'(\bar{x}-c\bar{t})$ has a discontinuity

then argue as above.

§1.4: 1, 2, 8



1. $\begin{cases} u_{tt} - c^2 u_{xx} = 0 \\ u(x,0) = \sin x \\ u_x(x,0) = 0 \\ u(0,t) = 0 \\ u_x(\pi/2, t) = 0 \end{cases}$

$f(\bar{x})$ is a $4l = 2\pi$ periodic funtion. $f(x) = \sin(x)$ fits this description for all values.

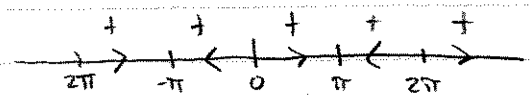
$u(x,t) = \frac{1}{2}[f(x+ct) + f(x-ct)] + 0 = \frac{1}{2}[\sin(x+ct) + \sin(x-ct)]$

$= \frac{1}{2}[\cancel{\cos(x)\sin(ct)} + \sin(x)\cos(ct) - \cancel{\cos(x)\sin(ct)} + \sin(x)\cos(ct)]$

✓ $\boxed{u(x,t) = \sin x \cos(ct)}$

2. $\begin{cases} u_{tt} - c^2 u_{xx} = 0 \\ u(x,0) = f(x) = 0 \\ u_x(x,0) = g(x) = \cos x \\ u_x(0,t) = u_x(\pi,t) = 0 \\ c = 1 \end{cases}$

free endpoints means that there should be even reflections about 0 and $l = \pi$.



$g(x)$ needs to be a function of period 2π with even reflections across all multiples of π. $g(x) = \cos x$ fits this for all x

$u(x,t) = 0 + \frac{1}{2}\int_{x-t}^{x+t} \cos\bar{x}\, d\bar{x}$

$= \frac{1}{2}[\sin(x+t) - \sin(x-t)]$

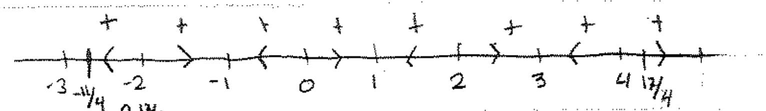
$= \frac{1}{2}[\cancel{\sin x \cos t} + \cos x \sin t - \cancel{\sin x \cos t} + \sin x \cos t]$

✓ $u(x,t) = \cos x \sin t.$

matho

8. $u(x,0) = f(x) = x^2(1-x)^2$ Find $u(3/4, 7/2)$

$u_t(x,0) = g(x) = 1$ $x-ct = -11/4$ $x+ct = 17/4$



$u_x(0,t) = u_x(1,t) = 0$

$c = 1$

$u(3/4, 7/2) = \frac{1}{2}\left(f(17/4) + f(-11/4)\right) + \frac{1}{2}\int_{-11/4}^{17/4} g(\bar{x})\,d\bar{x}$

$= \frac{1}{2}\left(f(1/4) + f(3/4)\right) + \frac{1}{2}\int_{-11/4}^{17/4} 1\,dx$

$= \frac{1}{2}\left[(1/4)^2(3/4)^2 + (3/4)^2(1/4)^2\right] + \frac{1}{2}\left[17/4 + 11/4\right] = \boxed{\frac{905}{256}}$

③

§1.5: 1-3

1. $\begin{cases} u_{tt} - c^2 u_{xx} = e^x \\ u(x,0) = f(x) = 0 \\ u_x(x,0) = g(x) = 0 \end{cases}$ Find $u(\frac{1}{2}, \frac{3}{2})$

$u(x,t) = 0 + 0 + \frac{1}{2c} \iint_{\triangle} F(\bar{x},\bar{t})\,d\bar{x}\,d\bar{t}$

no bar

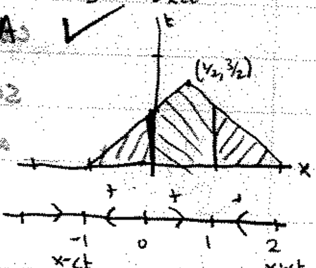
$$u(x,t) = \frac{1}{2c}\int_0^t \int_{x-c(t-\bar{t})}^{x+c(t-\bar{t})} F(\bar{x},\bar{t})\,d\bar{x}\,d\bar{t}$$
$$= \frac{1}{2c}\int_0^t \int_{x-c(t-\bar{t})}^{x+c(t-\bar{t})} e^{\bar{x}}\,d\bar{x}\,d\bar{t}$$
$$= \frac{1}{2c}\int_0^t e^{\bar{x}+c(t-\bar{t})} - e^{\bar{x}-c(t-\bar{t})}\,d\bar{t}$$
$$= \frac{e^x}{2c}\left[\int_0^t e^{c(t-\bar{t})}\,d\bar{t} - \int_0^t e^{-c(t-\bar{t})}\,d\bar{t}\right]$$
$$= \frac{e^x}{2c}\left[\frac{-1}{c}\left[e^{c(t-t)} - e^{c(t)}\right] - \frac{1}{c}\left(e^{-c(t-t)} - e^{-c(t)}\right)\right]$$
$$= \frac{1}{2c^2}e^x\left[-e^0 - e^0 + e^{ct} + e^{-ct}\right]$$
$$= \boxed{\frac{1}{c^2}e^x\left[\cosh(ct) - 1\right]}$$

✓ So $u(\frac{1}{2}, \frac{3}{2}) = \square$ @ $x = \frac{1}{2}$, $t = \frac{3}{2}$.

2. $\begin{cases} u_{tt} - u_{xx} = e^x & \text{for } 0<x<1 \quad t>0 \\ u(x,0) = 0 & 0<x<1 \\ u_t(x,0) = 0 & 0\le x<1 \\ u_x(0,t) = 0 \\ u_x(1,t) = 0 \end{cases}$ ✓ find $u(\frac{1}{2}, \frac{3}{2})$

$u(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\bar{x})\,d\bar{x} + \frac{1}{2c}\iint_{\triangle} F(\bar{x},\bar{t})\,dA$

$= \iint_{\triangle} F(\bar{x},\bar{t})\,dA$ ✓



even ✓

✓

$$u(x,t) = \frac{1}{2}\left[\int_{-1}^0\int_0^{\bar{x}+1} e^{-\bar{x}}\,d\bar{t}\,dx + \int_0^{\frac{1}{2}}\int_0^{\bar{x}+1} e^{\bar{x}}\,d\bar{t}\,d\bar{x} + \int_{\frac{1}{2}}^1\int_0^{2-\bar{x}} e^{\bar{x}}\,d\bar{t}\,d\bar{x} + \int_1^2\int_0^{2-\bar{x}} e^{(2-\bar{x})}\,d\bar{t}\,d\bar{x}\right]$$
$$= \frac{1}{2}\left[\int_{-1}^0 (\bar{x}+1)e^{-\bar{x}}\,d\bar{x} + \int_0^{\frac{1}{2}} (\bar{x}+1)e^{\bar{x}}\,d\bar{x} + \int_{\frac{1}{2}}^1 (2-\bar{x})e^{\bar{x}}\,d\bar{x} + \int_1^2 (2-\bar{x})\,d\bar{x}\right]$$
$$= \frac{1}{2}\left[\int_{-1}^0 -\bar{x}e^{-\bar{x}}\,d\bar{x} + \int_{-1}^0 e^{-\bar{x}}\,d\bar{x} + \int_0^{\frac{1}{2}} \bar{x}e^{\bar{x}}\,d\bar{x} + \int_0^{\frac{1}{2}} e^{\bar{x}}\,d\bar{x} + 2\int_{\frac{1}{2}}^1 e^{\bar{x}}\,d\bar{x} - \int_{\frac{1}{2}}^1 \bar{x}e^{\bar{x}}\,d\bar{x} + \int_1^2 (2-\bar{x})x^{2-\bar{x}}\,d\bar{x}\right]$$

$u = -\bar{x}$, $du = -d\bar{x}$; $u = 2-\bar{x}$, $du = -d\bar{x}$

$$= \frac{1}{2}\left[\cancel{\int_1^0 ue^u\,du} + \int_{-1}^0 e^{-x}\,dx + \int_0^{\frac{1}{2}} xe^x\,dx + \int_0^{\frac{1}{2}} e^x\,dx + 2\int_{\frac{1}{2}}^1 e^x\,dx - \int_{\frac{1}{2}}^1 xe^x\,dx - \cancel{\int_1^0 ue^u\,du}\right]$$
$$= \frac{1}{2}\left[-\left[e^{-x}\right]_{-1}^0 + \left[(x-1)e^x\right]_0^{\frac{1}{2}} + \left[e^x\right]_0^{\frac{1}{2}} + 2\left[e^x\right]_{\frac{1}{2}}^1 - \left[(x-1)e^x\right]_{\frac{1}{2}}^1\right]$$
$$= \frac{1}{2}\left[-(e^0 - e^1) + (-\tfrac{1}{2}e^{\frac{1}{2}} - -1e^0) + (e^{\frac{1}{2}} - e^0) + 2(e^1 - e^{\frac{1}{2}}) - (0e^1 - -\tfrac{1}{2}e^{\frac{1}{2}})\right]$$
$$= \frac{1}{2}\left[\cancel{-1} + e - \cancel{\tfrac{1}{2}e^{\frac{1}{2}}} + \cancel{1} + e^{\frac{1}{2}} - 1 + 2e - 2e^{\frac{1}{2}} - \cancel{\tfrac{1}{2}e^{\frac{1}{2}}}\right]$$
$$= \boxed{\boxed{\frac{1}{2}\left[3e - 2e^{\frac{1}{2}} - 1\right]} = u(\tfrac{1}{2}, \tfrac{3}{2})}$$

✓

3. $$\begin{cases} u_{tt} - u_{xx} = \sin \pi x & \text{for } 0 < x \le 1 \quad t > 0 \\ u(x,0) = 0 = f(x) & \text{for } 0 \le x \le 1 \\ u_t(x,0) = 0 = g(x) & 0 \le x \le 1 \\ u(0,t) = u(1,t) = 0 & \end{cases}$$

$$u(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\bar{x})\,d\bar{x} + \frac{1}{2c}\iint_{\Delta} F(\bar{x},\bar{t})\,dA$$

$$u(x,t) = 0 + 0 + \frac{1}{2}\int_0^t \int_{x-(t-\bar{t})}^{x+(t-\bar{t})} \sin(\pi \bar{x})\,d\bar{x}\,d\bar{t}$$

$$= \frac{-1}{2\pi}\int_0^t \left(\cos(\pi x + \pi(t-\bar{t})) - \cos(\pi x - \pi(t-\bar{t}))\right) d\bar{t}$$

$$= -\frac{1}{2\pi}\int_0^t \left[\cancel{\cos(\pi x)\cos(\pi t - \pi\bar{t})} - \sin(\pi x)\sin(\pi t - \pi \bar{t}) - \cancel{\cos(\pi x)\sin(\pi t - \pi\bar{t})} - \sin(\pi x)\sin(\pi t - \pi\bar{t})\right] d\bar{t}$$

$$= \frac{\sin(\pi x)}{\pi}\int_0^t \sin(\pi t - \pi\bar{t})\,d\bar{t}$$

$$= \frac{\sin(\pi x)}{\pi^2}\left[\cos(\pi t - \pi\bar{t})\right]_0^t$$

$$= -\frac{\sin(\pi x)}{\pi^2}\left[\cos(0) - \cos(\pi t)\right]$$

$$= \boxed{\frac{\sin(\pi x)}{\pi^2}\left[\cos(\pi t) - 1\right]}$$

✓

could also get this by superposition, starting with a steady state sol'n.

Andy Ibarra
Math 5440
HW 2
due 9/3/10

Class exercise 1: Prove that a continuous differentiable function $f(x)$ $(x \in \mathbb{R})$ is even iff $f'(x)$ is odd and odd iff $f'(x)$ is even.

Proof: • Suppose $f'(x)$ is odd. Then $f'(-x) = -f'(x)$. By the Fundamental Theorem of Calculus,

$$f(x) = f(0) + \int_0^x f'(t)\,dt.$$

Now, $f(-x) = f(0) + \int_0^{-x} f'(t)\,dt$ Let $u = -t$, $du = -dt$

$$= f(0) + \int_0^x f'(-u)(-du)$$

$$= f(0) + \int_0^x -f'(u)(-du)$$

$$= f(0) + \int_0^x f'(u)\,du$$

$= f(x)$, so $f(x)$ is even.

Suppose now that $f(x)$ is even. Then $\forall a \in \mathbb{R}$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(-a) = \lim_{h \to 0} \frac{f(-a+h) - f(-a)}{h}$$

$$= \lim_{h \to 0} \frac{f(a-h) - f(a)}{h} \quad \text{(since } f \text{ is even)}$$

Let $k = -h$, then $k \to 0$ as $h \to 0$, so

$$f'(-a) = \lim_{k \to 0} \frac{f(a+k) - f(a)}{-k}$$

$$= -\lim_{k \to 0} \frac{f(a+k) - f(a)}{k}$$

$= -f'(a)$, so $f'(x)$ is odd.

yes!

- Suppose $f'(x)$ is even and $f(0)=0$.

$$f(x) = f(0) + \int_0^x f'(t)\,dt$$

Thus $f(-x) = f(0) + \int_0^{-x} f'(t)\,dt$ let $u=-t$, $du=-dt$

$$= f(0) + \int_0^{x} f'(-u)(-du)$$

$$= f(0) - \int_0^{x} f'(u)\,du$$

$$= f(0) - [f(x) - f(0)]$$

$$= 2f(0) - f(x)$$

$$= -f(x). \Rightarrow f(x) \text{ is odd.}$$

Suppose $f(x)$ is odd. Then, for each $a \in \mathbb{R}$,

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$f'(-a) = \lim_{h\to 0} \frac{f(-a+h)-f(-a)}{h}$$

$$= \lim_{h\to 0} \frac{-f(a-h)+f(a)}{h}$$

let $k=-h$, then $k\to 0$ as $h\to 0$, so

$$f'(-a) = \lim_{k\to 0} \frac{-f(a+k)+f(a)}{-k}$$

$$= \lim_{k\to 0} \frac{f(a+k)-f(a)}{k}$$

$$= f'(a)$$

so $f'(x)$ is even.