

Math 4200-001
Week 4-5 concepts and homework
1.6, 2.1-2.2
Due Friday September 25 at 11:59 p.m.

1.6 10, 14

2.1 2ac, 3, 5, 10, 11, 13, 14;

2.2 1ad, 2 (prove with FTC!), 3 (work in reverse to rewrite as a contour integral that you can evaluate), 4, 6, 8, 9, 10 (use the antiderivative theorem and slightly modify Example 1.6.8).

Hint: In many of these problems the fundamental theorem of Calculus for contour integrals lets you find the answer very quickly once you find an antiderivative on an appropriate domain.

w4.1 (extra credit) This is a careful version of 1.6.6. Part (a) is relatively straightforward. I consider part (b) to be challenging.

a) Solve $\sin(z) = w$ for z using the quadratic formula and logarithms. Keep careful track of the multi-valued nature of the inverse sine function $\arcsin(z)$. Note that the quadratic formula yields two solutions except when $\cos(z) = 0$.

b) Prove that there is a branch of $\arcsin(z)$ defined on the branch domain we used in class for $\sqrt{z^2 - 1}$, namely $\mathbb{C} \setminus \{x \in \mathbb{R} \text{ s.t. } |x| \geq 1\}$, which is a bijection to the vertical strip $\left\{x + iy \mid -\frac{\pi}{2} < x < \frac{\pi}{2}\right\}$. This branch extends the Calculus $\arcsin(x)$

which was defined as a differentiable function on the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$.