

## A Power Law For Human Heights and Weights

### Body Mass Index

In this part of your project you will use the method of least squares to find a power law which (approximately) relates human weights to heights. There is something called the "Body Mass Index", or BMI, which is not a great scientific index but which is often mentioned in the context of the American overweight problem. It is a crude measure of whether someone is overweight or underweight, and whether they are at risk for certain bad health outcomes. There are many sites which let you compute your B.M.I., and which tell you a little bit about it.

A person's BMI is computed by dividing their weight by the square of their height, and then multiplying by a universal constant. (If you measure weight in kilograms, and height in meters, this constant is the number one.) Thus, the assumption underlying BMI is that for adults at equal risk levels (but different heights), weight should be proportional to the square of height. It is easy to deduce that if people were to scale equally in all directions when they grew, weight would scale as the cube of height. That particular power law seems a little high, since adults don't look like uniformly expanded versions of babies; we seem to get relatively stretched out when we grow taller. (Although it may not be a bad power if you eliminate babies from your analysis.) One would expect the best predictive power to be somewhere between 2 and 3. If the power is much larger than 2 then one could argue that the body mass index might need to be modified to reflect this fact. Of course, the sample heights and weights which Utah ACCESS students collected this summer may or may not be representative of a healthy population, but let's proceed anyway. (In previous years our experimental power has always come out between 2.35 and 2.65. The national baby–young adult data in the accompanying file yielded a power of 2.59.)

To find "our" power law we will use data collected by the ACCESS 2009 class, here at Utah. You will do a linear regression fit to the ln–ln data, as we discussed for the planetary orbit radius vs. period fit. The data is in a file on our project 2 page.

### Project 2a Assignment

Create a document using Maple in which you go through the process of finding the best power law to fit your collected height–weight data. In particular,

- 1) Find a least squares line fit to the ln–ln data which you obtain from the ACCESS data. Make sure that you use the "map(evalf,V)" command on your vectors of ln(hts) and ln(wts), before trying to do the least squares computation. Otherwise Maple will try to work symbolically rather than with decimals, and will fail.
- 2) Create a plot display which shows the ln–ln data and your least squares line fit.
- 3) Deduce the optimal power law for our (ACCESS) height–weight data. Create a single plot display which shows the graph of the power function and the pointplot of our original data.
- 4) What does our power law predict for the weights of "average" people, at heights of 5 feet, 5.5 feet, 6 feet, and 6.5 feet?
- 5) On the internet, find the "normal" BMI range (for adults), i.e. the upper and lower "normal" BMI numbers, using the inch–pound system. Since in these units,

$$BMI = \frac{703 w}{h^2}$$

the graph of heights and weights for which BMI has a constant value B is the parabola

$$w = \frac{B h^2}{703}.$$

Create a SINGLE display containing

- a) the 475 ACCESS data points

b) the national power law function, from 1976–1980 medians.

c) the ACCESS power law function from the 2009 data.

d) The upper and lower parabolas for the "normal" BMI range.

Use different colors to identify the various points and curves. Make a key/legend on your plot to identify each component (there is such a plot option.) Our power law, the national power law, and the BMI curves may all fit the adult data pretty well. If you copy and paste the display into a text field then you won't lose the legend if you re-execute the display command later by accident.

6) Explain, using your display from (5), why most children don't fit into the "normal" BMI range, based on our calculated power law and how it compares to a power law with  $p=2$ . How are the BMI standards adjusted for children? Is this consistent with our data?

7) Compare the national 1976–1980 power law for kids and young adults to the data and power law from 2009. Does it appear that people are heavier now, for given heights?

*Note: It turns out a Belgian demographer, Adolphe Quetelet, also called the "Father of Statistics", originally proposed a power of  $p=2$  for adults, based on his own data analysis during the early 1800's. In a footnote which history has forgotten, he said that a power of 2.5 is more appropriate if you want an approximation for people of all ages.*