Math 2250-010

Week 2 concepts and homework, due January 17.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems. There will also be a Maple/software assignment posted this week. It won't require you to hand in any work, but there will be Maple work due on the following Friday. We will run Maple introduction sessions, times TBA. See our homework page for details.

<u>1.2.40</u>, postponed from last week.

1.3 Slope fields and solution curves: understand how the graph of the solution to a first order DE IVP is related to the underlying slope field; the existence-uniqueness theorem for solutions to IVPs.

1.3: 11,12, <u>13</u>, <u>14</u>, 17, 18, <u>24</u>. (Use "dfield" for 24, see link below.)

w2.1) Consider the initial value problem in 1.3.11 above, namely

$$y'(x) = 2 x^2 y''$$

 $y(1) = -1.$

<u>a</u>) Use the existence-uniqueness theorem to show that there some open interval containing $x_0 = 1$ on

which this initial value problem has a unique solution.

b) The differential equation in this problem is separable, so you can actually find a solution to the initial value problem above. Do so.

<u>c)</u> What is the largest interval on which the solution to <u>b</u> is defined as a differentiable function? Explain. <u>d</u>) Use the *dfield* Java tool at the URL below, to plot the part of the initial value problem solution graph that is contained in the rectangle -1 < x < 3, -5 < y < 3. The URL (which you can always find by searching for "dfield") is

http://math.rice.edu/~dfield/dfpp.html

w2.2) In homework problem 1.1.29 last week you showed that if a function y = y(x) has the property that every straight line normal to the graph y = y(x) passes through the point (0, 1), then y(x) satisfies the differential equation

$$\frac{dy}{dx} = -\frac{x}{y-1}$$

a) Use the existence-uniqueness theorem to verify that the initial value problem for this differential equation, with y(3) = -3, has a unique solution.

b) Use dfield to create a slope field for this differential equation inside the rectangle

-6 < x < 6, -5 < y < 7. What do you notice about the apparent shapes of the solution graphs? Does this make sense based on the original geometric description that led to this differential equation? Explain. **c)** Solve the initial value problem in part **a**, to explicitly find y(x). What is the largest *x*-interval on which y(x) can be defined as a differentiable function? Explain. 1.4: 2,3,<u>4</u>,9,<u>12</u>,13,19,<u>20</u>,21: solving DE's and IVP's for separable differential equations <u>36</u>, 41, <u>42</u>, <u>45</u>, <u>49</u>, <u>50</u>, <u>54</u>: modeling and solving problems with first order separable DEs. <u>Note:</u> The correct answer to 42 is 4.25 billion years, not the 1.25 billion years that appears in the back of the book.

1.5: <u>1</u>, 7, <u>8</u>, 13, <u>24</u>, solving linear DE's and IVP's.

<u>w2.3</u>) In last week's homework problem $\underline{w1.3}$ you were asked to check that the functions

$$y(x) = x + C e^{-x}$$

are solutions to the differential equation

$$y'(x) = x - y + 1$$

Use the methods of section 1.5 to find these same solutions, that you were told last week before you knew how to solve linear differential equations.