Math 2250-010
Week 1 concepts and homework, due Friday January 10 (at the start of class).

All of the indicated problems are good for seeing if you can work with the underlying concepts. The underlined problems are to be handed in. The quiz at the start of class on Friday will be drawn from all of these concepts and from these or related problems.

1.1 Check whether a given function solves a ***differential equation****. If solutions to a first order DE are given with a "free" constant C, find which C value solves a given* ***initial value problem****; translate geometric or modeling properties described in words into differential equations satisfied by the solution functions. Combine the above ideas to solve more complicated problems.*

1.1: 1, 4, 5, **6**, 9, **15**, 19, **27**, **29**, **30**, 32, 33, **34**.
w1.1) Consider the first order differential equation for functions $y(x)$:

$$y' = -3\,y + 9\,x\ .$$

a) Show that functions $y(x) = 3\,x - 1 + C\,e^{-3\,x}$ solve this DE, where C is any constant.
b) Solve the initial value problem for this DE, with $y(0) = 4$.

1.2 Differential equations $y'(x) = f(x)$ which can be solved by direct antidifferentiation $y(x) = \int f(x)dx + C$: Solve such DE's using Calculus techniques. Solve for particle velocity and postion, given a formula for the acceleration function. Solve for position if velocity is described graphically. Applications.

1.2: 1, **2**, 5, **6**, **7**, **9**, 10, 13, 15, **16**, **18**, 21, 22, **24** (except make the building 800 feet high rather than 400 feet high) **26** (except make the initial velocity 50 $\frac{m}{s}$ instead of 100 $\frac{m}{s}$), 31, 32, 33, **40**. **(postpone 40 until next week).**
w1.2) Solve the following initial value problems as a way to review important integration techniques from Calculus: substitution and integration by parts.

a) $\dfrac{dy}{dx} = 4\sin\left(\dfrac{x}{2}\right),\ y(0) = 2$.

b) $y'(x) = 2\,x\,e^{-x},\ y(0) = 0$.

c) $\dfrac{dy}{dx} = \dfrac{4\,x}{\sqrt{x^2+9}},\ y(0) = 6$.

1.3 Slope fields and solution curves: understand how the graph of the solution to a first order DE IVP is related to the underlying slope field.

1.3: **2**, 3, 5, **6**, 10. For **2**, **6**, you may just xerox the book's slope fields. Alternately you may choose to google the applet "dfield", which has dialog boxes that you use, in order to have the software draw these slope fields for you. Then you can take screen shots and print them out. I should have time to demonstrate this in class on Wednesday. (There is also a Matlab version of dfield, which you can google, download, and use inside Matlab.)

w1.3) Consider the differential equation in 1.3.6, namely

$$y'(x) = x - y + 1.$$

a) Show that the functions $y(x) = x + C\,e^{-x}$ solve this differential equation.

b) Find the value of C in the general solution above, so that $y(x)$ solves the initial value problem

$$y'(x) = x - y + 1$$
$$y(-1) = 2.$$

Identify the graph of this solution on your slope field from **6**. Also add the diagonal asymptote for this graph, and write its equation.