Name_____

Student I.D._____

Math 2250-1
Quiz 8 Solutions
October 26, 2012

1) Consider the differential equation for $x(t)$, which could arise in a model for mechanical motion:

$$x''(t) + 4x'(t) + 5x(t) = 0 .$$

1a) Find the general solution to this differential equation.

(5 points)

The characteristic polynomial is

$$p(r) = r^2 + 4r + 5 = (r+2)^2 + 1.$$

The roots satisfy

$$(r+2)^2 + 1 = 0 \Rightarrow (r+2)^2 = -1 \Rightarrow r + 2 = \pm i \Rightarrow r = -2 \pm i .$$

You could also find these with the quadratic formula:

$$r^2 + 4r + 5 = 0 \Rightarrow r = -\frac{4 \pm \sqrt{16 - 20}}{2} = -2 \pm \frac{\sqrt{-4}}{2} = -2 \pm i .$$

So the homogeneous solution is

$$x(t) = c_1 e^{-2t}\cos(t) + c_2 e^{-2t}\sin(t) .$$

1b) Which damping phenomenon is exhibited by solutions to this differential equation?

(1 point)

underdamped.

1c) Use your work in (1a) to solve the initial value problem

$$x''(t) + 4x'(t) + 5x(t) = 0$$
$$x(0) = 2$$
$$x'(0) = 0 .$$

(4 points)

$$x(t) = c_1 e^{-2t}\cos(t) + c_2 e^{-2t}\sin(t)$$
$$x'(t) = c_1\left(-2e^{-2t}\cos(t) + e^{-2t}(-\sin(t))\right) + c_2\left(-2e^{-2t}\sin(t) + e^{-2t}\cos(t)\right) .$$

So,

$$x(0) = 2 = c_1$$
$$x'(0) = 0 = -2c_1 + c_2$$

So $c_1 = 2$, $c_2 = 4$. *So*

$$x(t) = 2e^{-2t}\cos(t) + 4e^{-2t}\sin(t) .$$