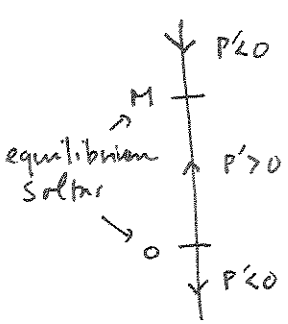
Math 2250-1
Tues 9/6

§2.1–2.2. Mostly we will use Friday's notes.

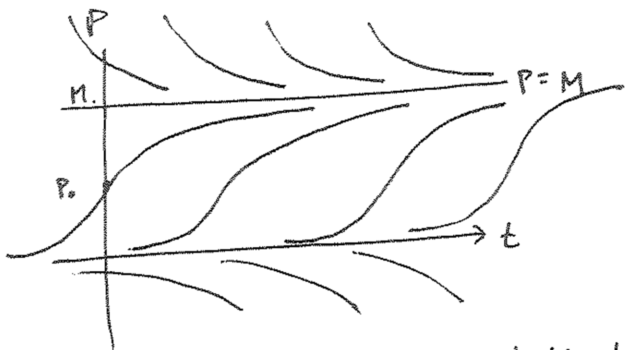
On Friday we talked about improved population models, which led to the logistic DE

$$\text{IVP} \begin{cases} \frac{dP}{dt} = kP(M-P) \\ P(0) = P_0 \end{cases}$$

We made predictions about solution behavior, based on the slope field and phase portrait



phase portrait



slope field, as represented by typical solution graphs.

We were part way through solving the IVP via separation of variables. This is as far as we'd gotten:

$$\frac{dP}{P(P-M)} = -k\,dt$$

$$\frac{1}{M}\left(\frac{1}{P-M} - \frac{1}{P}\right)dP = -k\,dt$$

← we used partial fractions, but you can save time by noticing

$$\frac{1}{P-a} - \frac{1}{P-b} = \frac{P-b-(P-a)}{(P-a)(P-b)} = \frac{a-b}{(P-a)(P-b)}$$

$$\int\left(\frac{1}{P-M} - \frac{1}{P}\right)dP = \int -Mk\,dt$$

$$\ln|P-M| - \ln|P| = -Mkt + C_1$$

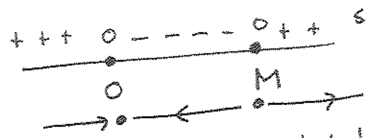
finish this! – see page 1 Friday and finish finding the sol'n.

More "real" examples (for if, and after we finish Friday's notes!)

doomsday extinction:

$$\frac{dP}{dt} = aP^2 - bP \qquad a,b>0.$$

$$= kP(P-M) \qquad \begin{matrix} k=a \\ -kM=b \end{matrix}$$



0 stable; M Unstable!

e.g. if $\beta(t)$ (fertility rate) $\sim P$ (proportional to)

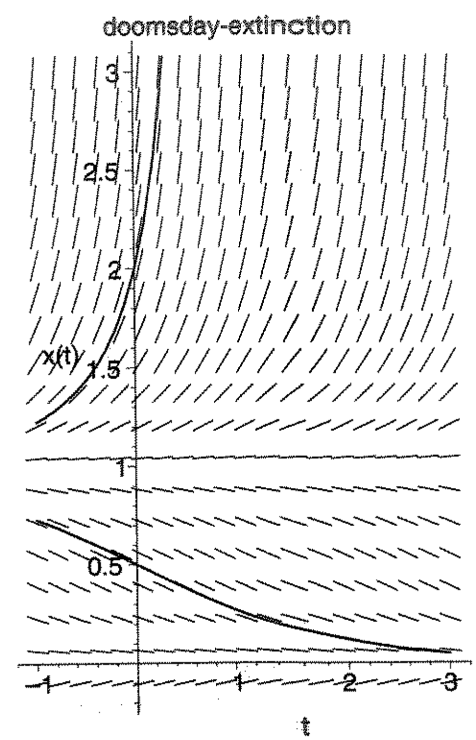
(e.g. herd roams in a fixed area and mates randomly upon meeting opposite-sex mate)

then $B(t) = aP^2$;

perhaps $D(t) = -bP$.

If $0 < P_0 < M$ then $\lim_{t \to \infty} P(t) = 0$ (so in real life, extinction)

If $P_0 > M$ then $\exists\ t_1 < \infty$ s.t. $\lim_{t \to t_1} P(t) = +\infty$ (doomsday!)



<u>example</u>

$$\begin{cases} \frac{dx}{dt} = x(x-1) \\ x(0) = 2 \end{cases}$$

Find the time of doomsday.

(ans $t = \ln 2$!)