Math 2250-1
Fri 9/23
§3.5-3.6

On Wed. we began studying matrix inverses

→ finish p. 4-5 Wed, including A^{-1} for $A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 5 & 0 \\ 2 & 7 & 1 \end{bmatrix}$.

Then try to find B^{-1} for $B = \begin{bmatrix} 1 & 5 & 5 \\ 2 & 5 & 0 \\ 2 & 7 & 4 \end{bmatrix}$

oh oh! Why can't B have an inverse matrix?

<u>Theorem</u> Let $A_{n\times n}$. Then A has an inverse matrix if and only if $\text{rref}(A) = I$

<u>proof</u>: A^{-1} should be the solution matrix X to

$$AX = I$$

i.e. $$A\left[\text{col}_1(X) \,\vdots\, \text{col}_2(X) \,\vdots\, \cdots \right] = \left[\begin{array}{c|c|c} 1 & 0 & \\ 0 & 1 & \cdots \\ \vdots & 0 & \\ 0 & \vdots & \end{array}\right]$$

find the columns of X by solving n linear systems at once, with an n-augmented matrix:

$$A \mid I$$

$$\Big\downarrow \text{row reduce!}$$

$$\text{rref}(A) \mid B$$

if $\text{rref}(A) = I$

$$I \,\vdots\, B$$

says the sol'n to

$$A\,\text{col}_j(X) = \vec{e}_j$$

is $\text{col}_j(B)$

that is, $X = B$!

$\Rightarrow AB = I$

If we wanted to see if also $BA = I$, we'd want to solve $BY = I$

$$B \,\vdots\, I$$

$$\Big\downarrow \text{reverse original row ops!}$$

$$I \,\vdots\, A \qquad \text{So } BA = I.$$

iff $\text{rref}(A) \neq I$, the bottom row of $\text{rref}(A)$ is zero

thus sol'ns to $A\vec{x} = \vec{b}$ don't always exist, and when they do, the sol'ns are not unique.

Thus A^{-1} does not exist

(because if it did, sol'ns to $A\vec{x} = \vec{b}$ is always $\vec{x} = A^{-1}\vec{b}$)

There's a nice formula for inverse of a 2×2 matrix:

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$ exists iff $D = ad-bc \neq 0$

(We call D the <u>determinant</u> of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and write $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$)

straight lines

for $D \neq 0$,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- check this formula works (we could have derived it with elementary row operations, but it's easy to check since we're told the formula).

- for a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the reduced row echelon form $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ if and only if the two rows are not multiples of each other. for $a,b,c,d \neq 0$, this is the ratio conditions $\frac{a}{c} \neq \frac{b}{d}$ i.e. $ad \neq bc$.

  this is also the correct condition if some entries $= 0$.

<u>example</u>: solve
$$3x + 7y = 5$$
$$5x + 4y = 8$$

On Monday we'll show how to compute determinants for larger square matrices. There's also a magic formula for A^{-1} when $A_{3\times3}$ or $A_{n\times n}$.