

linear system of m eqns in n unknowns:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ a_{31}x_1 + \dots & \\ \vdots & \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

rewritten in matrix form:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

i.e.

$$A \vec{x} = \vec{b}$$

A = coefficient matrix
 \vec{b} = RHS vector

in other words,

$$\begin{aligned} \text{entry}_i(A \vec{x}) &:= a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \\ &= \text{row}_i(A) \cdot \vec{x} \quad (\text{dot product}). \end{aligned}$$

[only works if the vector \vec{x} has as many entries as A has columns]

exercise: $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} =$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$

Check and record two important algebra properties

$$A(\vec{x} + \vec{y}) = A\vec{x} + A\vec{y}$$

$$A(k\vec{x}) = kA\vec{x}$$

k a scalar

Summarize our discussion yesterday about criteria for solvability of

$$A\vec{x} = \vec{b}$$

in terms of r.r.e.f. (A)

Here's a "typical" $\text{rref}(A)$ to motivate discussion

$$\begin{bmatrix} 1 & * & 0 & 0 & * & 0 \\ 0 & 0 & 1 & 0 & * & 0 \\ 0 & 0 & 0 & 1 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- $A\vec{x} = \vec{b}$ always has sol'tns \vec{x} if and only if ("iff")
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 i.e. no matter what \vec{b} is

- Solutions \vec{x} to $A\vec{x} = \vec{b}$ are unique iff
 ↑
 i.e. only one sol'n

- If $A_{n \times n}$ is a square matrix

then $\text{rref}(A) = I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} \Rightarrow$

$\text{rref}(A) \neq I \Rightarrow$

Exercise 1: Consider the 2nd order DE for $y(x)$:

$$y'' + 4y' + 3y = 6$$

(a) Show $y(x) = c_1 e^{-x} + c_2 e^{-3x} + 2$ solves this DE

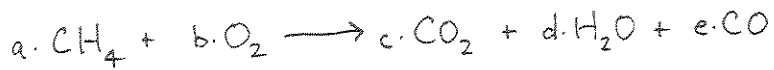
(b) Find values for c_1, c_2 to solve the IVP for this DE, with

$$y(0) = 4$$
$$y'(0) = 1$$

(c) What if the initial values were $y(0) = y_0$?
 $y'(0) = v_0$

(just to indicate breadth of possible applications)

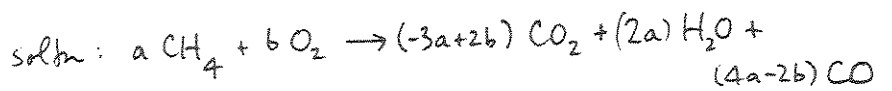
2) Balance the following combustion equation. Methane, CH₄, is the primary component of natural gas, and we've all heard about the dangers of carbon monoxide which ^{are} created if there is not enough O₂.



2a) Show $a = c + e$
 $4a = 2d$
 $2b = 2c + d + e$

2b) Order the variables c, d, e, a, b & write the linear system in synthetic form. Compute reduced row echelon form to solve for c, d, e in terms of a, b.

2c) Discuss implications!



We've been noticing a relationship between solutions of homogeneous matrix eqns and non-homogeneous ones. This actually just depends on $A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y}$ and $A(c\vec{x}) = cA\vec{x}$

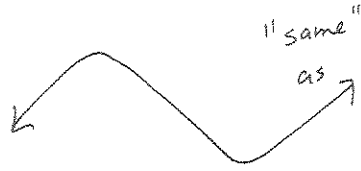
Theorem The general solution to

$$A\vec{x} = \vec{b}$$

is of the form $\vec{x} = \vec{x}_p + \vec{x}_H$
↑
(any) particular solution

general solution to the homogeneous equation $A\vec{x} = \vec{0}$

proof: If $A\vec{x}_p = \vec{b}$ and $A\vec{x}_H = \vec{0}$
then $A(\vec{x}_p + \vec{x}_H) = A\vec{x}_p + A\vec{x}_H = \vec{b} + \vec{0} = \vec{b}$



Let $L(y) := y' + p(x)y$
 $L(y_1 + y_2) = L(y_1) + L(y_2)$
 $L(cy) = cL(y)$
Check!

So same conclusion holds, to solve $L(y) = q(x)$,

i.e. $y' + p(x)y = q(x)$
gen'l sol'n is $y = y_p + y_H!$

If \vec{x} is any soltn to $A\vec{x} = \vec{b}$

then $\vec{x} = \vec{x}_p + (\vec{x} - \vec{x}_p)$
so $A\vec{x} = A[\vec{x}_p + (\vec{x} - \vec{x}_p)] = A\vec{x}_p + A(\vec{x} - \vec{x}_p)$

so $\vec{b} = \vec{b} + A(\vec{x} - \vec{x}_p)$
so $\vec{0} = A(\vec{x} - \vec{x}_p)$

so $\vec{x} - \vec{x}_p$ is a sol'n to the homog. eqn, i.e. $\vec{x} - \vec{x}_p = \vec{x}_H$

example 1

$$y' + 3y = 6$$
$$e^{3x}(y' + 3y) = 6e^{3x}$$
$$(e^{3x}y)' = 6e^{3x}$$
$$e^{3x}y = 2e^{3x} + C$$
$$y = 2 + \underbrace{C}_{y_H} e^{-3x}$$

↑ ↑ !!
y_p y_H

example 2

Exercise 1, page 3 today.