Math 2250-1
Monday 9/19

On Friday we used Gaussian elimination to find all sol'ns to

$$\begin{aligned} x_1 - 2x_2 + 3x_3 + 2x_4 + x_5 &= 10 \\ 2x_1 - 4x_2 + 8x_3 + 3x_4 + 10x_5 &= 7 \\ 3x_1 - 6x_2 + 10x_3 + 6x_4 + 5x_5 &= 27 \end{aligned}$$

↓ synthetic version

$$\left[\begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 2 & -4 & 8 & 3 & 10 & 7 \\ 3 & -6 & 10 & 6 & 5 & 27 \end{array}\right]$$

↓ Gaussian elimination, stage 1 (working to the right and down)

$$\left[\begin{array}{ccccc|c} 1 & -2 & 3 & 2 & 1 & 10 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{array}\right]$$

row echelon form (not unique, i.e. a given matrix can have different row echelon forms)

1. all zero rows at bottom
2. leading non-zero entry in each row is strictly to the right of the previous rows

stands for

$$\begin{aligned} x_1 - 2x_2 + 3x_3 + 2x_4 + x_5 &= 10 \\ x_3 \qquad + 2x_5 &= -3 \\ x_4 - 4x_5 &= 7 \end{aligned}$$

backsolve:

$x_5 = t$
$x_4 = 7 + 4t$
$x_3 = -3 - 2t$
$x_2 = s$
$x_1 = 10 - t - 2(7+4t) - 3(-3-2t) + 2s = 5 - 3t + 2s$

↓ Gaussian elimination, stage 2 (working up and to the left!)

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 3 & 5 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{array}\right]$$

reduced row echelon form (unique!!)

1, 2, and
3. each non-zero row leads with a 1, called the leading 1
4. each column with one of the row leading 1's, has zeroes in every other column entry.

stands for

$$\begin{aligned} x_1 - 2x_2 \qquad + 3x_5 &= 5 \\ x_3 \quad + 2x_5 &= -3 \\ x_4 - 4x_5 &= 7 \end{aligned}$$

backsolve

$x_5 = t$
$x_4 = 7 + 4t$
$x_3 = -3 - 2t$
$x_2 = s$
$x_1 = 5 - 3t + 2s$

Once you're comfortable with the reduced row echelon form algorithm use technology!

in vector form:

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -3 \\ 7 \\ 0 \end{bmatrix} + s\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ -2 \\ 4 \\ 1 \end{bmatrix} \quad ; s, t \in \mathbb{R}.$$