Math 2250-1
Friday 9/16
§3.1-3.2

- Do page 6 examples on Wed notes, after the overview on this page.

Linear system of m equations in n unknowns:

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots &+ a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots &+ a_{2n}x_n = b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots &+ a_{mn}x_n = b_m \end{aligned}\right\} LS$$

Goal: find all (unknown) solution vectors $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ (for which all m eqtns hold simultaneously)

the collection of these solutions is called the solution set

The rectangle of coefficients

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$ is called the coefficient matrix A

If we adjoin the right-hand side vector:

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & & a_{mn} & b_m \end{array}\right]$$ this is the augmented matrix $A|\vec{b}$

We may do the following equation operations without changing the solution set:

1. Interchange 2 eqtns
2. multiply an eqtn by a non-zero constant
3. replace an eqtn by its sum with any multiple of another eqtn

[we can combine several of these into a single step, e.g. replace an eqtn with the sum of a non-zero mult of it, and a multiple of another eqtn]

So when we work synthetically with the augmented matrix we may do the corresponding 3 elementary row operations without changing sol'n set:

1. interchange 2 rows ("swap")
2. multiply a row by a non-zero constant
3. replace a row by its sum with a multiple of another row.

Do a big example: Find all solutions to the system

$$
\begin{aligned}
x_1 - 2x_2 + 3x_3 + 2x_4 + x_5 &= 10 \\
2x_1 - 4x_2 + 8x_3 + 3x_4 + 10x_5 &= 7 \\
3x_1 - 6x_2 + 10x_3 + 6x_4 + 5x_5 &= 27.
\end{aligned}
$$

- work synthetically, with the augmented matrix
- Use Gaussian elimination to first get an equivalent augmented matrix in row-echelon form
  1. all "zero rows" (having all entries = 0) lie beneath the non-zero rows
  2. the leading non-zero entry in each non-zero row lies strictly to the right of the leading non-zero entry in the row above it.

  [at this stage you could "backsolve" to find all sol'ns].
- Continue with Gaussian elimination until matrix is in reduced row-echelon form: 1, 2, and
  3. each leading non-zero entry of a non-zero row is a "1", called a "leading 1"
  4. each column that has (a row's) leading 1 has zeroes for each other column entry.

$$
\left[\begin{array}{ccccc|c}
1 & -2 & 3 & 2 & 1 & 10 \\
2 & -4 & 8 & 3 & 10 & 7 \\
3 & -6 & 10 & 6 & 5 & 27
\end{array}\right]
$$

Class example for Friday September 16. Reduced row echelon form of matrices is unique, unlike row–echelon form. Good software implements Gaussian elimination to do this computation:

```
> with(LinearAlgebra) : #Maple linear algebra library
> A := Matrix(3, 5, [1,-2, 3, 2, 1, 2,-4, 8, 3, 10, 3,-6, 10, 6, 5]) :
    #our matrix of coefficients with 3 rows and 5 columns
  b := Vector(3, [10, 7, 27]) : #right hand side column vector
  Aaugb := <A|b>; # the augmented matrix
  Answer := ReducedRowEchelonForm(Aaugb);
```

$$Aaugb := \begin{bmatrix} 1 & -2 & 3 & 2 & 1 & 10 \\ 2 & -4 & 8 & 3 & 10 & 7 \\ 3 & -6 & 10 & 6 & 5 & 27 \end{bmatrix}$$

$$Answer := \begin{bmatrix} 1 & -2 & 0 & 0 & 3 & 5 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & -4 & 7 \end{bmatrix} \tag{1}$$

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