Math 2250-1
Wed Oct 5

finish p. 3-4 Tuesday

then notice that if $\{\vec{v}_1, \vec{v}_2, .. \vec{v}_k\}$ span the subspace W,
you can do the following "elementary vector operations"
to get other spanning sets:

(1) interchange $\vec{v}_j$ and $\vec{v}_\ell$
(2) replace $\vec{v}_j$ by $c\vec{v}_j$, $c \neq 0$:
because $c_1\vec{v}_1 + c_2\vec{v}_2 + .. + c_j\vec{v}_j + .. + c_k\vec{v}_k = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + \frac{c_j}{c}(c\vec{v}_j) + \dots + c_k\vec{v}_k$

(3) replace $\vec{v}_j$ with $\vec{v}_j + c\vec{v}_\ell$, $\ell \neq j$:

because $c_1\vec{v}_1 + .. + c_j\vec{v}_j + .. + c_\ell\vec{v}_\ell + \dots + c_k\vec{v}_k$
$= c_1\vec{v}_1 + .. + c_j(\vec{v}_j + c\vec{v}_\ell) + \dots + (c_\ell - cc_j)\vec{v}_\ell + .. c_k\vec{v}_k$

Exercise 14 Use elementary vector operations,
starting with the 6 columns of $A_{4\times 6}$ in Exercise 13,
to find an "optimal" basis for the span of these 6 columns.
(You're computing the reduced column echelon form of A!)

Exercise 15 One of your bases in Exercise 13 was

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

express the vectors in this basis as linear combos of the basis you found in Exercise 14, for the same subspace.

Exercise 16 What are the dimensions of

- the solution space to $A\vec{x} = \vec{0}$ in # 12?
- the column space of A, in # 13-14?

Exercise 17 Complete the basis in #15 for W, to a basis for $\mathbb{R}^4$. Hint: ~~add~~ augment with vectors not in the span of what you've got, until you span all of $\mathbb{R}^4$.

<u>Exercise 18</u> Use matrix theory to explain why

a) $n > m$ vectors in $\mathbb{R}^m$ must be linearly dependent

b) $n < m$ vectors in $\mathbb{R}^m$ cannot span $\mathbb{R}^m$.

c) Thus every basis of $\mathbb{R}^m$ consists of exactly m vectors.
(so dimension $= m$, and doesn't depend on choice of basis

d) What's your favorite basis of $\mathbb{R}^m$?

Exercise 18 is a special case of a more general fact for vector spaces (including subspaces).

<u>Theorem</u>: Let W be a vector space.

1) If n vectors span W, then any collection of $> n$ vectors must be dependent.

2) If n vectors in W are linearly indep., then no collection of $< n$ vectors can span W

3) If one basis of W has n vectors, every basis of W has n vectors.

(because $> n \Rightarrow$ dep (1)
$< n \Rightarrow$ can't span (2))

also

4) If $\dim(W) = n$ and n vectors in W are indep., then they also span so are basis

5) If $\dim(W) = n$ and n vectors span W, then they are also indep., so are basis.

<u>Exercise 19</u>: Consider the 2-d plane $3x - 2y + 5z = 0$.
Find a basis