Math 2250-1
Tuesday 10/4

§ 4.1-4.3 cont'd; start 4.4

Vocabulary/concept review:

<u>linear combination</u> of $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k$:

<u>span</u>$\{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k\}$:

$\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k$ <u>linearly independent</u>

" <u>linearly dependent</u>

<u>vector space</u> V

<u>subspace</u> $W \subset V$, where V is a vector space (<u>subspace</u> is not the same as <u>subset</u>).

Then finish pages 4-5 of Monday's notes, exercises 5-9

<u>Exercise 10</u> : Characterize all possible subspaces of $\mathbb{R}^3$ - there are 4 types

n exercises 9-10, we used (& understood):

Theorem: Let $\vec{v}_1, \dots \vec{v}_k$ be vectors in $\mathbb{R}^n$ (or V).
Then $\text{span}\{\vec{v}_1, \dots \vec{v}_k\}$ is a subspace of V

check

α) if $\vec{v} = c_1\vec{v}_1 + \dots + c_k\vec{v}_k$ and $\vec{u} = d_1\vec{v}_1 + \dots + d_k\vec{v}_k$ are both in $\text{span}\{\vec{v}_1, \dots \vec{v}_k\}$
then $\vec{v} + \vec{u} = (c_1 + d_1)\vec{v}_1 + \dots + (c_k + d_k)\vec{v}_k \in \text{span}\{\vec{v}_1, \dots \vec{v}_k\}$.

& β) $t\vec{v} = tc_1\vec{v}_1 + tc_2\vec{v}_2 + \dots + tc_k\vec{v}_k \in \text{span}\{\vec{v}_1, \dots \vec{v}_k\}$. ∎

ercise 6 is a special case of

Theorem Let $A_{m\times n}$. The solution set to the homogeneous matrix equation $A\vec{x} = \vec{0}$
i.e. $W := \{\vec{x} \in \mathbb{R}^n \text{ s.t. } A\vec{x} = \vec{0}\}$ is a subspace of $\mathbb{R}^n$

check

α) if $\vec{x}, \vec{y} \in W$, then $A\vec{x} = \vec{0}$ and $A\vec{y} = \vec{0}$. Thus $A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y} = \vec{0} + \vec{0} = \vec{0}$
So $\vec{x} + \vec{y} \in W$.

β) if $\vec{x} \in W$, then $A\vec{x} = \vec{0}$ so
$$A(c\vec{x}) = cA\vec{x} = c\vec{0} = \vec{0}$$
so $c\vec{x} \in W$. ∎

Definition: If $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k$ span a subspace W
and if $\vec{v}_1, \ldots \vec{v}_k$ are linearly independent,
then $\{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_k\}$ is called a basis for W.

Definition: The number of vectors in a basis for a subspace W is called the dimension of W

Exercise 11
What are the dimensions of the 4 subspace types in exercise 10?

```
> with(LinearAlgebra):
> A := Matrix(4, 6, [1, 2, 0, 1, 1, 2,
                     2, 4, 1, 4, 1, 7,
                     -1, -2, 1, 1, -2, 1,
                     -2, -4, 0, -2, -2, -4]);
```

$$A := \left[\begin{array}{cccccc|c} 1 & 2 & 0 & 1 & 1 & 2 & 0 \\ 2 & 4 & 1 & 4 & 1 & 7 & 0 \\ -1 & -2 & 1 & 1 & -2 & 1 & 0 \\ -2 & -4 & 0 & -2 & -2 & -4 & 0 \end{array}\right]$$

```
> ReducedRowEchelonForm(A);
```

$$\left[\begin{array}{cccccc|c} 1 & 2 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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>
```

Exercise 12 Find a basis for the solution space to $A\vec{x} = \vec{0}$, for the $A_{4\times6}$ at right. Explain why your basis spans the solution space. Verify that your basis is linearly independent. (The solution set to $A\vec{x} = \vec{0}$ is called the nullspace of A.)

Exercise 13: Another natural subspace associated to the matrix $A_{4\times 6}$ in Ex. 12, is the span of the 6 columns. This is a subspace of $\mathbb{R}^4$. (called the column space). How many different bases for this subspace can you find, using only (some) of the columns of the original matrix?

Hint: if you begin with all 6 columns and successively discard columns which are dependent on the ones you have left, you won't decrease their span.... keep doing this until you have an independent spanning set.

```
> with(LinearAlgebra) :
> A := Matrix(4, 6, [1, 2, 0, 1, 1, 2,
                     2, 4, 1, 4, 1, 7,
                     -1, -2, 1, 1, -2, 1,
                     -2, -4, 0, -2, -2, -4]);
```

$$A := \left[\begin{array}{cccccc|c} 1 & 2 & 0 & 1 & 1 & 2 & 0 \\ 2 & 4 & 1 & 4 & 1 & 7 & 0 \\ -1 & -2 & 1 & 1 & -2 & 1 & 0 \\ -2 & -4 & 0 & -2 & -2 & -4 & 0 \end{array}\right]$$

```
> ReducedRowEchelonForm(A);
```

$$\left[\begin{array}{cccccc|c} 1 & 2 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

```
>
```