Math 2250−1

Tuesday October 25

Graphs illustrating undamped, overdamped, underdamped, and critically damped mass−spring systems − following the examples on Tuesday's notes.

1)

$$x''(t) + 9\cdot x(t) = 0$$
$$x(0) = 1$$
$$x'(0) = \frac{3}{2} .$$

```
> with(DEtools) :
> with(plots) :
> deqtn1 := diff(x(t), t, t) + 9·x(t) = 0;
  ICS := x(0) = 1, D(x)(0) = 3/2;
  dsolve({deqtn1, ICS}, x(t));
```

$$deqtn1 := \frac{d^2}{dt^2} x(t) + 9\, x(t) = 0$$

$$ICS := x(0) = 1, D(x)(0) = \frac{3}{2}$$

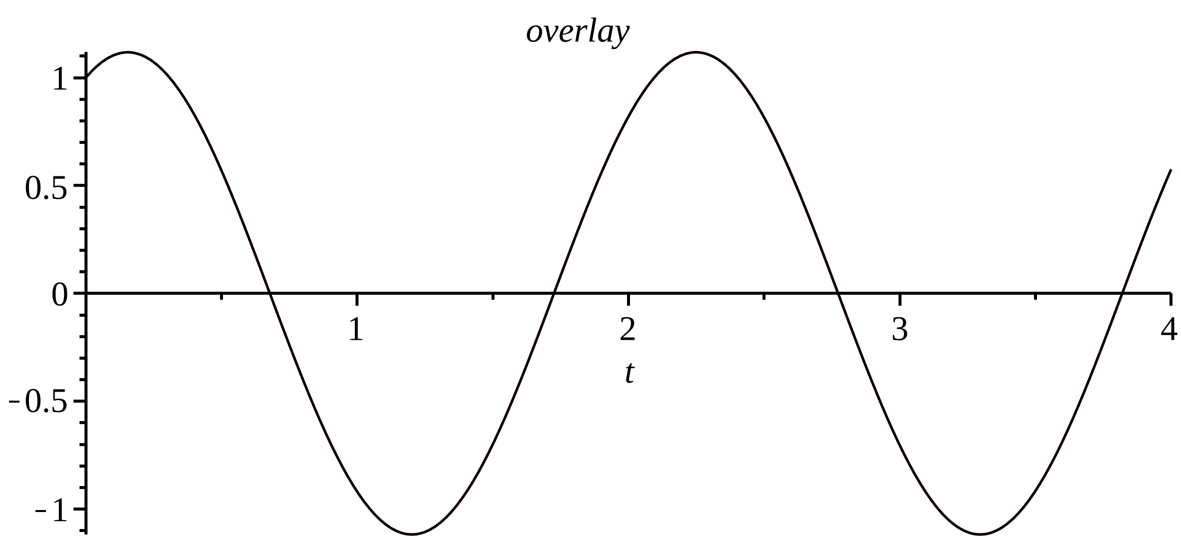
$$x(t) = \frac{1}{2}\sin(3\,t) + \cos(3\,t) \quad \textbf{(1)}$$

```
> C := sqrt(1 + .5^2);
  alpha := arctan(.5);
```

$$C := 1.118033989$$

$$\alpha := 0.4636476090 \quad \textbf{(2)}$$

```
> plot1a := plot(1/2 ·sin(3· t) + cos(3· t), t = 0 ..4, color = red) :
  plot1b := plot(C·cos(3·t − alpha), t = 0 ..4, color = black) :
  display({plot1, plot2}, title = ‘overlay‘);
```




```
>
```

2a)

$$x''(t) + 6\cdot x'(t) + 9\cdot x(t) = 0$$
$$x(0) = 1$$
$$x'(0) = \frac{3}{2}.$$

```
> deqtn2a := diff(x(t), t, t) + 6·diff(x(t), t) + 9·x(t) = 0;
  dsolve({deqtn2a, ICS}, x(t));
```

$$deqtn2a := \frac{d^2}{dt^2} x(t) + 6 \left(\frac{d}{dt} x(t)\right) + 9\, x(t) = 0$$

$$x(t) = e^{-3t} + \frac{9}{2} e^{-3t} t \quad (3)$$

```
>
```

2b)

$$x''(t) + 10\cdot x'(t) + 9\cdot x(t) = 0$$
$$x(0) = 1$$
$$x'(0) = \frac{3}{2}.$$

```
> deqtn2b := diff(x(t), t, t) + 10·diff(x(t), t) + 9·x(t) = 0;
  dsolve({deqtn2b, ICS}, x(t));
```

$$deqtn2b := \frac{d^2}{dt^2} x(t) + 10 \left(\frac{d}{dt} x(t)\right) + 9\, x(t) = 0$$

$$x(t) = \frac{21}{16} e^{-t} - \frac{5}{16} e^{-9t} \quad (4)$$

```
>
```

2c)

$$x''(t) + 2\cdot x'(t) + 9\cdot x(t) = 0$$
$$x(0) = 1$$
$$x'(0) = \frac{3}{2}.$$

```
> deqtn2c := diff(x(t), t, t) + 2·diff(x(t), t) + 9·x(t) = 0;
  dsolve({deqtn2c, ICS}, x(t));
```

$$deqtn2c := \frac{d^2}{dt^2} x(t) + 2 \left(\frac{d}{dt} x(t)\right) + 9\, x(t) = 0$$

$$x(t) = \frac{5}{8} \sqrt{2}\, e^{-t} \sin\left(2\sqrt{2}\, t\right) + e^{-t} \cos\left(2\sqrt{2}\, t\right) \quad (5)$$

$$> C1 := \mathrm{sqrt}\left(\frac{50.0}{64} + 1\right);$$

$$\alpha 1 := \arctan\left(\frac{5.0\cdot \mathrm{sqrt}(2.0)}{8.0}\right);$$

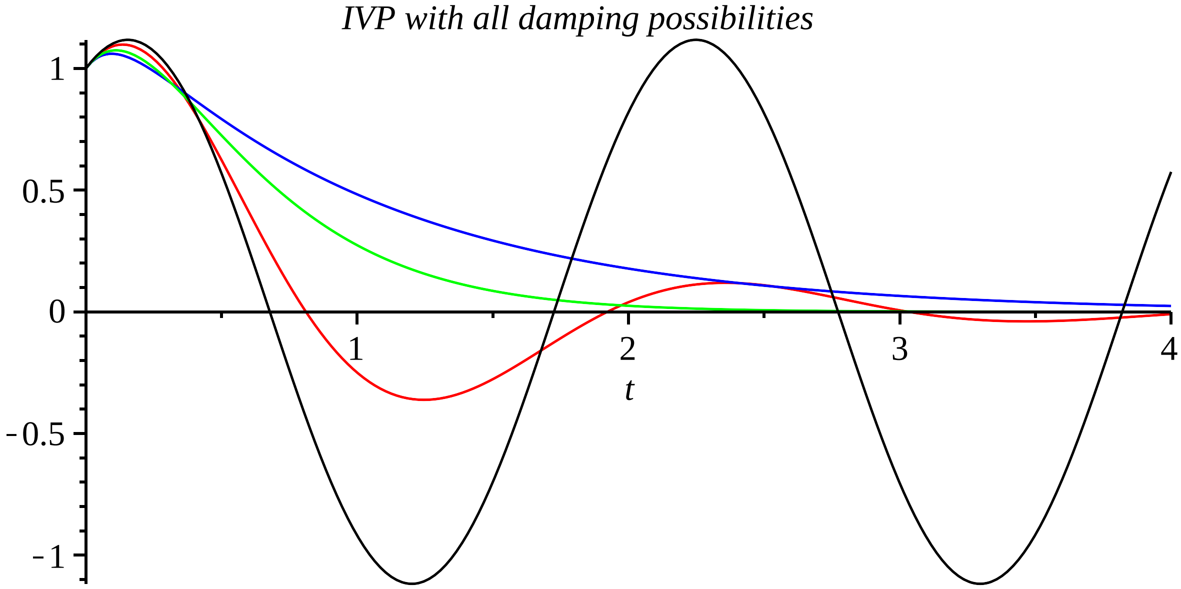
$$C1 := 1.334634782$$

$$\alpha 1 := 0.7238392540 \quad (6)$$

>

All four solutions at once:

> $plot2a := plot\left(\exp(-3\cdot t)\cdot\left(1+\frac{9}{2}\cdot t\right), t=0..4, color=green\right):$
$plot2b := plot\left(\frac{21}{16}\cdot\exp(-t)-\frac{5}{16}\cdot\exp(-9\cdot t), t=0..4, color=blue\right):$
$plot2c := plot(C1\cdot\exp(-t)\cdot\cos(2\cdot\text{sqrt}(2)\cdot t-\alpha 1), t=0..4, color=red):$
$display(\{plot1b, plot2a, plot2b, plot2c\}, title=\text{`IVP with all damping possibilities`});$



>