Math 2250-1
Fri 10/21

review &
finish p. 2-3 Wed (we were essentially done.)
then begin §5.3, after this superposition page

other illustrations of $y = y_P + y_H$ for linear operators L:

<u>Exercise 1</u> Consider the linear equation

$$x_1 - 2x_2 + 3x_3 = 7$$

show the general solution is of the form $\vec{x} = \vec{x}_p + \vec{x}_H$

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

special case of
$L(\vec{x}) = A\vec{x}$
and the general soln to
$A\vec{x} = \vec{b}$
is $\vec{x} = \vec{x}_p + \vec{x}_H$.

<u>Exercise 2</u> Consider the 1st order linear DE for $y(x)$

$$y' + 5y = 10$$

solve for $y = y_P + y_H$ and compare to Chapter 1 sol'n.

$$L(y) = y' + 5y$$

§5.3 How to solve constant coefficient homogeneous linear DE's

$$L(y) := y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_1 y' + a_0 y = 0 \qquad a_j\text{'s constant.}$$

step 1 try $y = e^{rx}$
then $L(y) = \underbrace{(r^n + a_{n-1}r^{n-1} + \dots + a_1 r + a_0)}_{p(r),\ \text{the characteristic polynomial}} e^{rx}$

So any root of $p(r)$ yields an exponential function solution

Now, there are several cases of increasing complexity.

Case I If $p(r)$ has n distinct (different) real roots $r_1, r_2, \dots r_n$
then $y_H(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x} + \dots + c_n e^{r_n x}$ is the general soltn.

(we've done lots of examples.)
i.e. $\{e^{r_1 x}, e^{r_2 x}, \dots e^{r_n x}\}$ is a basis for the n-dim'l soltn space.

Exercise 3 : Check the Wronskian in cases $n = 2, 3 \rightarrow$ an interesting pattern emerges,
which can be proven with induction, i.e.
google "Vandermonde matrix"

Case II Repeated roots (real roots)

- If $(r-\alpha)^k$ is a factor of $p(r)$, then

  $e^{\alpha x}, \; xe^{\alpha x}, \ldots \, x^{k-1}e^{\alpha x}$ are k linearly ind. sol'ns.

- If $p(r) = (r-r_1)^{k_1}(r-r_2)^{k_2}\cdots(r-r_j)^{k_j}$ $\quad k_1+k_2+\ldots+k_j=n$

  then a basis of solutions is given by

$$\Big\{ \underbrace{e^{r_1 x}, xe^{r_1 x}, \ldots x^{k_1-1}e^{r_1 x}}_{k_1 \text{ fcns}}, \underbrace{e^{r_2 x}, xe^{r_2 x}, \ldots x^{k_2-1}e^{r_2 x}}_{k_2 \text{ fcns}}, \ldots \underbrace{e^{r_j x}, xe^{r_j x}, \ldots x^{r_j-1}e^{r_j x}}_{k_j \text{ fcns}} \Big\}$$

altogther, n independent functions that solve the DE!

Exercise 4 : Find the general sol'n to $y'''' - y''' = 0$
by first finding $v = y'''$ $\quad (v' - v = 0)$.
Verify that in this case your answer agrees with the algorithm above

the text has a good explanation of why case II algorithm works, see page 316-318 "polynomial differential operators"

Case III Complex roots : Monday!!