Math 2250-1
Tues 10/18

- finish Monday's notes, p. 4-5; get comfortable with Wronskians

- Exercises 2, 3 both illustrated:

  Consider the constant coefficient, homogeneous linear DE for $y(x)$:

  $*$ $ay'' + by' + cy = 0$. $(a \neq 0)$

  If r_1, r_2 are distinct roots of the characteristic polynomial
  $$p(r) = ar^2 + br + c$$
  then $e^{r_1 x}, e^{r_2 x}$ are a basis for
  solns to $*$

Why do $e^{r_1 x}, e^{r_2 x}$ solve the DE?
Why are they a basis?

<u>Exercise 4</u> Find a basis for the solution space to

$$y'' + 4y' + 4y = 0$$

(notice $r = -2$ is a double root to $p(r) = r^2 + 4r + 4$ ...)

<u>Exercise 5</u> consider $y'' + by' + cy = 0$

where $p(r) = r^2 + br + c = (r - r_1)^2$

(so r_1 is a double root, and
$b = -2r_1$
$c = r_1^2$).

Show $y_1(x) = e^{r_1 x}$
$y_2(x) = x e^{r_1 x}$
are a basis for the solution space.

Exercise 6 Consider the DE

$$y'' + y = 0.$$

a) Show $\cos x$, $\sin x$, $\cos(x - \pi/4)$, $\sin(x - \pi/4)$ all solve this DE.

b) Show that the last two functions are linear combo's of the first 2. (There must be dependencies since the soltn space is only 2-dimensional.

c) Solve $\begin{cases} y'' + y = 0 \\ y(0) = 3 \\ y'(0) = 4 \end{cases}$

Solve $\begin{cases} y'' + y = 0 \\ y(\pi/4) = 3 \\ y'(\pi/4) = 4. \end{cases}$

d) What's the characteristic polynomial for this DE ??

Higher order linear DE's follow the pattern of 2nd order ones.

- Soltns to IVP

$$\text{IVP}\begin{cases} y^{(n)} + p_{n-1}(x)\,y^{(n-1)} + \cdots + p_1(x)\,y' + p_0(x)\,y = f(x) \\ y(a) = b_0 \\ y'(a) = b_1 \\ \vdots \\ y^{(n-1)}(a) = b_{n-1} \end{cases}$$

  exist and are unique on on any interval I, as long as $a \in I$, and $p_{n-1}(x), \ldots p_1(x), p_0(x), f(x)$ are continuous on I

- The solution space to the homogeneous linear DE
$$y^{(n)} + p_{n-1}(x)\,y^{(n-1)} + \ldots + p_1(x)\,y' + p_0(x)\,y = 0$$
  is an n-dimensional subspace.

Exercise 7 : Find all solutions to $y''' + 3y'' - y' - 3y = 0$

hint: try exponential fcns for a basis.