Math 2250-1
Tues 11/8

Session to go over last spring's midterm exam is
today 4-5:30 pm
WEB L102

Complete pages 3-4 of Monday's notes.

then,

<u>exercise 1</u> Use Laplace transform and partial fractions to solve for $x(t)$:

$$\begin{cases} x''(t) + 4x(t) = 8te^{2t} \\ x(0) = 0 \\ x'(0) = 1 \end{cases}$$

use this example to make sure you understand partial fractions.
we'll check our work on Maple

Laplace transforms and partial fractions

```
> with(inttrans);
```

[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable]

```
> X := s -> 8/((s^2+4)·(s-2)^2) + 1/(s^2+4);
```

$$X := s \to \frac{8}{(s^2+4)\,(s-2)^2} + \frac{1}{s^2+4}$$

```
> invlaplace(X(s), s, t);
```

$$\frac{1}{2}\cos(2t) + \frac{1}{2}\sin(2t) + \frac{1}{2}e^{2t}(2t-1)$$

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> convert(X(s), parfrac, s);
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$$\frac{1}{2}\,\frac{s+2}{s^2+4} + \frac{1}{(s-2)^2} - \frac{1}{2\,(s-2)}$$

notice how the Laplace transform table is constructed to make use of partial fraction decomposition terms.

<u>exercise 2</u> Use the table to compute

$$\mathcal{L}^{-1}\left\{ \frac{3s-6}{s^2+2s+5} - \frac{7}{s^2+2s+1} + \frac{5}{(s^2+1)^2} + \frac{4s}{(s-2)^2} \right\}(t)$$

Table of Laplace Transforms

This table summarizes the general properties of Laplace transforms and the Laplace transforms of particular functions derived in Chapter 10.

Function	Transform	Function	Transform
$f(t)$	$F(s) = \int_0^\infty e^{-st} f(t)\,dt$	e^{at}	$\dfrac{1}{s-a}$
$af(t) + bg(t)$	$aF(s) + bG(s)$	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
$f'(t)$	$sF(s) - f(0)$	$\cos kt$	$\dfrac{s}{s^2+k^2}$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$	$\sin kt$	$\dfrac{k}{s^2+k^2}$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	$\cosh kt$	$\dfrac{s}{s^2-k^2}$
$\int_0^t f(\tau)\,d\tau$	$\dfrac{F(s)}{s}$	$\sinh kt$	$\dfrac{k}{s^2-k^2}$
$e^{at}f(t)$	$F(s-a)$	$e^{at}\cos kt$	$\dfrac{s-a}{(s-a)^2+k^2}$
$\cancel{u(t-a)f(t-a)}$	$\cancel{e^{-as}F(s)}$	$e^{at}\sin kt$	$\dfrac{k}{(s-a)^2+k^2}$
$\cancel{\int_0^t f(\tau)g(t-\tau)\,d\tau}$	$\cancel{F(s)G(s)}$	$\dfrac{1}{2k^3}(\sin kt - kt\cos kt)$	$\dfrac{1}{(s^2+k^2)^2}$
$tf(t)$	$-F'(s)$	$\dfrac{t}{2k}\sin kt$	$\dfrac{s}{(s^2+k^2)^2}$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	$\dfrac{1}{2k}(\sin kt + kt\cos kt)$	$\dfrac{s^2}{(s^2+k^2)^2}$
$\dfrac{f(t)}{t}$	$\int_s^\infty F(\sigma)\,d\sigma$	$\cancel{u(t-a)}$	$\cancel{\dfrac{e^{-as}}{s}}$
$\cancel{f(t),\ \text{period } p}$	$\cancel{\dfrac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)\,dt}$	$\cancel{\delta(t-a)}$	$\cancel{e^{-as}}$
1	$\dfrac{1}{s}$	$\cancel{(-1)^{[\![t/a]\!]}\ \text{(square wave)}}$	$\cancel{\dfrac{1}{s}\tanh\dfrac{as}{2}}$
t	$\dfrac{1}{s^2}$	$\cancel{[\![\tfrac{t}{a}]\!]\ \text{(staircase)}}$	$\cancel{\dfrac{e^{-as}}{s(1-e^{-as})}}$
t^n	$\dfrac{n!}{s^{n+1}}$		
$\cancel{\dfrac{1}{\sqrt{\pi t}}}$	$\cancel{\dfrac{1}{\sqrt{s}}}$		
$\cancel{t^a}$	$\cancel{\dfrac{\Gamma(a+1)}{s^{a+1}}}$		