Math 2250-1
Fri 11/4

HW for next week 10.1–10.3 due Wed
exam 2 is Thurs.

§ 10.1–10.2 Laplace transforms
Continue filling in Laplace transform table and solving initial value problems

exercise 1 (to review)

$$\mathcal{L}\{7 - 5\cos 3t + 2\sin 8t\}(s) =$$

$$\mathcal{L}^{-1}\left\{\frac{2}{s+3} + \frac{4}{s^2+9}\right\}(t) =$$

exercise 2 Solve the IVP for $x(t)$, that we rushed through part of on Wed...

$$\text{IVP}\begin{cases} x''(t) + 4x(t) = 10\cos 3t \\ x(0) = 2 \\ x'(0) = 1 \end{cases}$$

$f(t)$	$F(s) := \int_0^\infty e^{-st} f(t)\,dt$, $s > M$.	
$f(t)+g(t)$	$F(s)+G(s)$	already checked
$cf(t)$	$cF(s)$	✓
⓪ 1	$1/s$	✓
⓪ t	$1/s^2$	
⓪ t^n	$n!/s^{n+1}$ $n \in \mathbb{N}$	
⓪ $e^{\alpha t}$	$1/s-\alpha$	✓
① $\cos kt$	s/s^2+k^2	✓
① $\sin kt$	k/s^2+k^2	✓
① $\cosh(kt)$	s/s^2-k^2	
① $\sinh(kt)$	k/s^2-k^2	
② $f'(t)$	$sF(s)-f(0)$	✓
② $f''(t)$	$s^2F(s)-sf(0)-f'(0)$	✓
② $f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$	
② etc.		
② $\int_0^t f(\tau)\,d\tau$	$F(s)/s$	
analogous! $tf(t)$	$-F'(s)$	
analogous! $t^2f(t)$	$F''(s)$	
analogous! $t^3f(t)$	$-F'''(s)$	
analogous! etc.		
analogous! $f(t)/t$	$\int_s^\infty F(\sigma)\,d\sigma$	
③ $e^{at}f(t)$	$F(s-a)$	
③ $u(t-a)$	e^{-as}/s	
③ $u(t-a)f(t-a)$	$e^{-as}F(s)$	
$e^{at}\cosh t$	$\frac{s-a}{(s-a)^2+k^2}$	
$e^{at}\sinh t$	$k/(s-a)^2+k^2$	
④ $t\cos kt$	$(s^2-k^2)/(s^2+k^2)^2$	
④ $\frac{1}{2k}t\sin kt$	$s/(s^2+k^2)^2$	
④ $\frac{1}{2k^3}(\sin kt - kt\cos kt)$	$1/(s^2+k^2)^2$	
④ te^{at}	$1/(s-a)^2$	

& more!

work down the table:

exercise 3 (0) $\mathcal{L}\{t^n\}(s) \quad n \in \mathbb{N}$

exercise 4 (1b) $\mathcal{L}\{\cosh(kt)\}(s)$
$\mathcal{L}\{\sinh(kt)\}(s)$

exercise 5 (2a) $\mathcal{L}\{f'''(t)\}(s)$
$\mathcal{L}\{f^{(n)}(t)\}(s)$
$\mathcal{L}\left\{\int_0^t f(\tau)\right\}(s)$

exercise 6 Use $\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\}(s) = \frac{F(s)}{s}$

to find $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+4)}\right\}(t)$.

(Notice you could also use partial fractions but it would take longer.)

exercise 7

(3c) $\mathcal{L}\{e^{at}\cos kt\}(s)$ via Euler's formula & $\mathcal{L}\{e^{(a+ik)t}\}(s)$

$\mathcal{L}\{e^{at}\sin kt\}(s)$

(3a) $\mathcal{L}\{e^{at}f(t)\}(s) = F(s-a)$ in general

((3c) is special case)

exercise 8 ~~damped~~ undamped forced oscillator; solve with Laplace transform

$$\begin{cases} x'' + 6x' + 34x = 0 \\ x(0) = 3 \\ x'(0) = 1. \end{cases}$$

ans:

$$x(t) = 3e^{-3t}\cos 5t + 2e^{-3t}\sin 5t$$