Math 2250-1
Fri 8/26

HW for next week will be posted by class time.

Today: separable DE's theory and examples (§1.4)
used to illustrate and understand general existence-uniqueness theorem for first order DE IVP's (§1.3)

• The algorithm for separable DE's, why it works, and assumptions:

DE separable means:

$$\frac{dy}{dx} = f(x)\phi(y) = \frac{f(x)}{g(y)} \qquad \left(\text{as long as } \phi(y) \neq 0;\ g(y) = \frac{1}{\phi(y)}\right)$$

legal math sol'n

$$g(y(x))\, y'(x) = f(x)$$

If $\int g(y)\,dy = G(y)$, $\int f(x)\,dx = F(x)$
are antiderivs of $g(y)$ & $f(x)$

$$\frac{d}{dx} G(y(x)) = \frac{d}{dx} F(x) \qquad \text{on an } x\text{-interval}$$

$$\Rightarrow\ G(y(x)) = F(x) + C$$

$$G(y) = F(x) + C$$

expresses $y(x)$ <u>implicitly</u> as a fcn of x.
you <u>may</u> be able to use algebra to get an explicit formula for y as a fcn of x

algorithm

$$g(y)\,dy = f(x)\,dx$$

$$\int g(y)\,dy = \int f(x)\,dx$$

$$G(y) = F(x) + C$$

same answer!

issues: if $\phi(y_0) = 0$ then algorithm fails ($g(y_0)$ not defined)
but you do get constant solutions
$$y(x) = y_0 \quad \forall x.$$
(since for this const. soltn, $\frac{dy}{dx} = 0$
$f(x)\phi(y_0) = 0$.

Exercise 0) Try this for the DE from 1.1.46

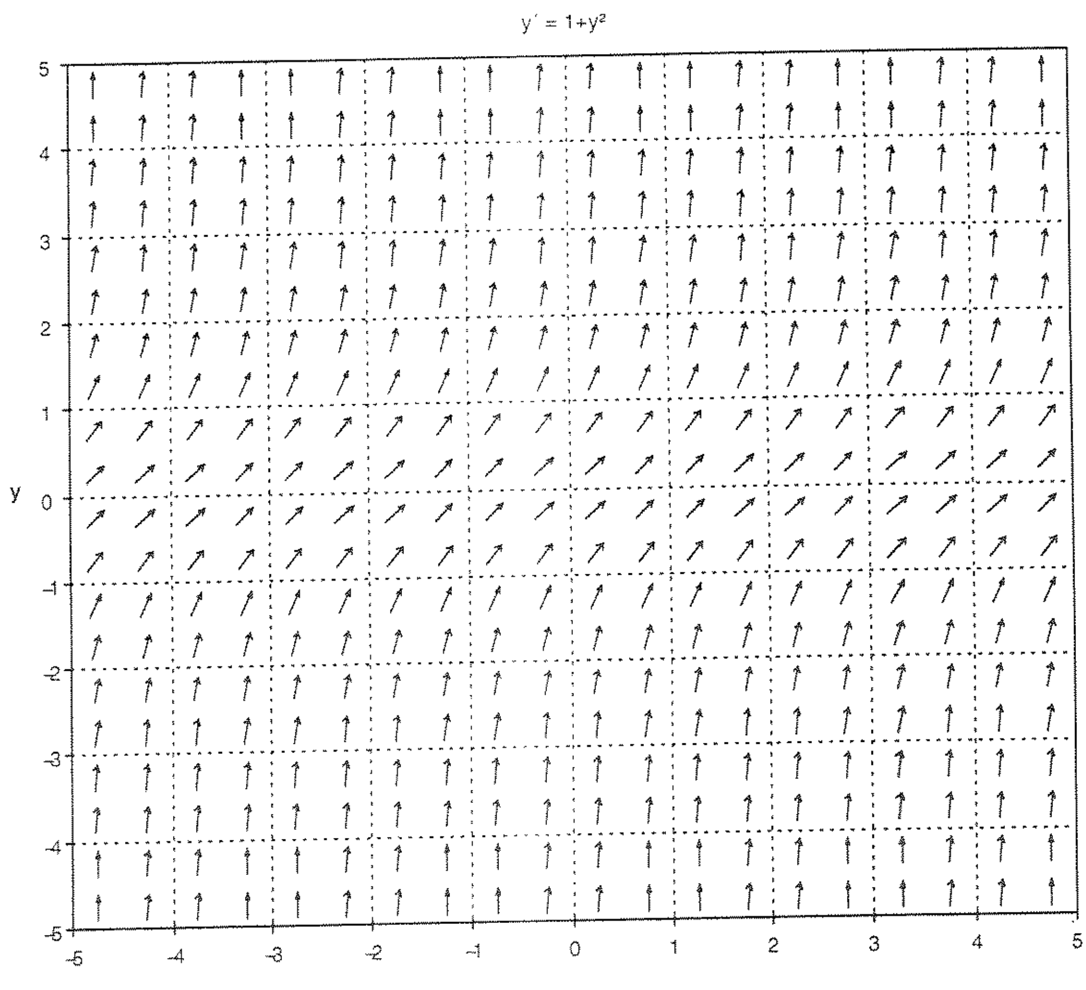
$$\frac{dv}{dt} = k v^2$$

Exercise 1a) Use sep. vars to solve

$$\frac{dy}{dx} = 1 + y^2$$

1b) sketch some soltn graphs onto slope field
(this is actually a quick way to describe the slope field if you know the soltns)

1c) explain why each IVP has a soltn, but this soltn does not exist for all x



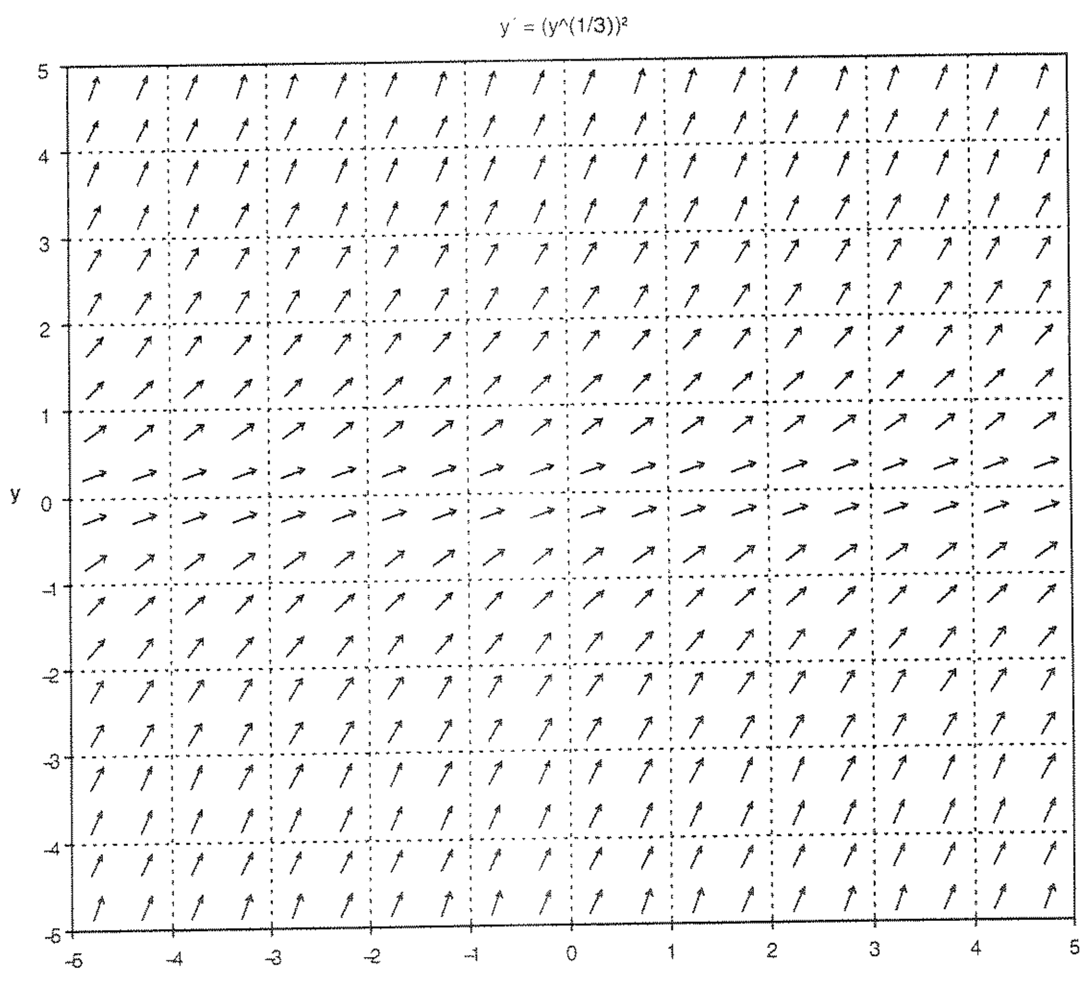
Exercise 2

2a) Solve $\begin{cases} \frac{dy}{dx} = y^{2/3} \\ y(0) = 0 \end{cases}$

use separation of variables

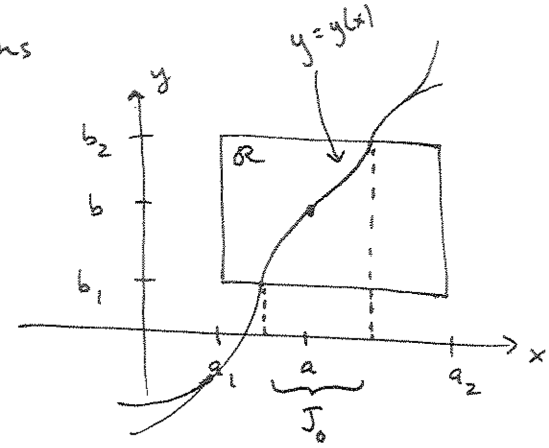
2b) But there are actually a lot more soltns to this IVP! (Ones you don't get from sep. var's are called <u>singular</u> soltns, see page 1.).

2c) Sketch some of these onto the slope field.



§1.3 : slope fields; existence and uniqueness for IVP's.

One reason we've been drawing slope fields is to motivate an existence and uniqueness theorem for 1st order DE initial value problems



<u>Theorem 1</u> Existence and uniqueness of solutions
$\hookrightarrow \exists$ $\hookrightarrow !$

Consider

$$\text{IVP} \begin{cases} \dfrac{dy}{dx} = f(x,y) \\ y(a) = b \end{cases}$$

Let the point (a,b) be interior to a coordinate rectangle $\mathcal{R}$ $\left(\begin{array}{l} a_1 \le x \le a_2 \\ b_1 \le y \le b_2 \end{array} \right)$

in the x-y plane

- Let $f(x,y)$ be continuous in $\mathcal{R}$
  Then
  $\exists$ sol'n to IVP on some interval J containing a in its interior
- If $\frac{\partial f}{\partial y}$ exists and is continuous in $\mathcal{R}$ then this solution is unique on any subinterval J_0 such that the solution graph lies inside $\mathcal{R}$

proof: Appendix A
intuition: "follow the slope field" to <u>get</u> sol'ns.
$\frac{\partial f}{\partial y}$ continuous turns out to guarantee that multiple solutions can't peel off!

<u>Exercise 3</u> : Discuss $\exists!$ theorem in previous examples. (where we used Calculus and already knew $\exists!$)

If time, dicuss $\exists!$ for

$$\begin{cases} \frac{dy}{dx} = 6x(y-1)^{2/3} \\ y(x_0) = x_0. \end{cases}$$