Math 2250-1
Wednesday 8/24

Recitation session tomorrow:
2250-6 meets Th. JFB 103
8:35-9:25

- Do example 4, page 5 Tuesday notes.

Then §1.3 **Slope fields & solution curves** using dfield applet, and by hand.

Consider

$$\text{IVP} \begin{cases} \frac{dy}{dx} = f(x,y) \\ y(x_0) = y_0 \end{cases}$$

and the graph of the solution function $y(x)$.

- This graph passes through (x_0, y_0).
- At every point (x,y) on the graph its slope $\left(\frac{dy}{dx}\right)$ is given by $f(x,y)$

Example 1a: Solve the §1.2 IVP

$$\text{IVP} \begin{cases} \frac{dy}{dx} = x-3 \\ y(1) = 2 \end{cases}$$

and sketch the solution graph on the interval $0 \le x \le 6$

For the differential equation

$$\frac{dy}{dx} = f(x,y)$$

the slope of any solution graph passing through (x,y) is determined by $f(x,y)$.
so, for each point (x,y) there is the slope $f(x,y)$.
this assignment of slope values for points is called a slope field (or direction field).

(analogous to vector field in Physics or multivariable calculus)

you can use pictures of slope fields to sketch graphs of solutions to DE's, even if you don't have formulas for the solutions

Example 1b • draw the slope field corresponding to the DE

$$\frac{dy}{dx} = x-3 \; ; \quad \text{i.e. } m(x,y) = x-3$$

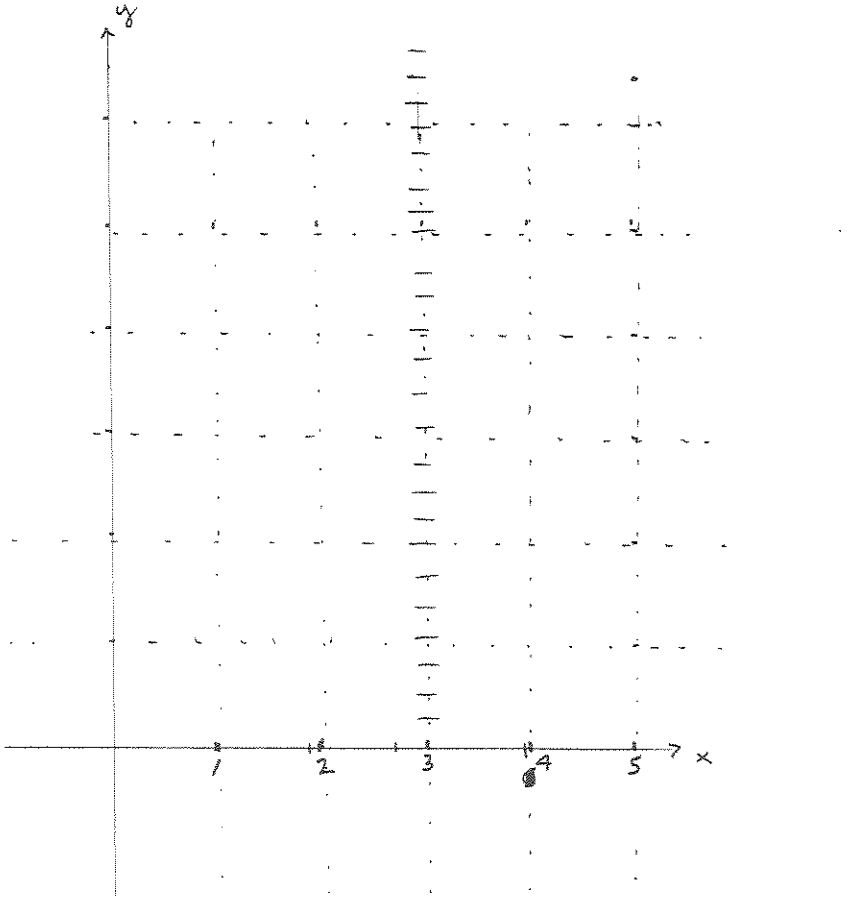
(use small line segments with appropriate slope)

slope value m	equation of corresponding isocline (curve on which slope is constant)
0	$x=3$

Then sketch soln to

$$\begin{cases} \frac{dy}{dx} = x-3 \\ y(1) = 2 \end{cases}$$

onto slope field picture
Compare with 1a.



This procedure can be automated!
google "dfield",
go to the top hit, and play with the Java application for drawing direction fields

Example 1c Use dfield to graph the solution graph to our

$$\text{IVP} \begin{cases} y' = x - 3 \\ y(1) = 2 \end{cases}$$

Example 2 Consider

$$\text{IVP} \begin{cases} \frac{dy}{dx} = y - x \\ y(0) = 0 \end{cases}$$

2a) have dfield draw the solution graph, say for $-3 \le x \le 3$, $-3 \le y \le 3$

2b) Check that
$y(x) = x + 1 + Ce^x$ solves the DE ← magic? (no; in §1.5 we'll learn how to derive this sol'n formula)

2c) find C to solve IVP.
Do your formulas agree with dfield graphs?

2d) Use isoclines to sketch a graph of the slope field by hand... use back of page.

an example of dfield output, for Example 2 ...