Math 2250-1
Tues 8/23

§1.2 : $y' = f(x)$

Recall from Monday, a 1st order DE is an equation

$$\boxed{F\left(x, y, \frac{dy}{dx}\right) = 0}$$

e.g. $\quad xy + (y')^2 = 0$

can usually use algebra to solve for y':

$$\boxed{\frac{dy}{dx} = f(x,y)}$$

If we also want to specify

$$y(x_0) = y_0$$

Then the DE together with the specified y-value ("initial value") is called an <u>i</u>nitial <u>v</u>alue <u>p</u>roblem

$$\boxed{\text{IVP} \begin{cases} \frac{dy}{dx} = f(x,y) \\ y(x_0) = y_0 \end{cases}}$$

§1.1 has a number of interesting examples which are worth reading through.

concepts (from Hw)

- check whether a given fcn solves a DE
- if sol'ns to 1st order DE are given with "free" const. C, find which C gives sol'n to IVP
- translate geometric or modeling properties described in words into differential equations satisfied by solution functions
- integrate the ideas above to solve more complex problems

§1.2 is for DE's $y' = f(x)$ which have sol'ns $y = \int f(x)dx + C$.

concepts (from Hw)

- solve these DE's by using Calculus integration techniques
- solve for velocity and position in physics-type problems, when acceleration is given with a formula
- solve for position if velocity is described graphically
- applications

Even if we don't always explicitly go over & list the main ideas in each section, you can recover them from how the text homework is organized.

Hw due <u>Fri</u> @ start of class!

<u>Example 1</u> (last example Monday)

Consider

$$\text{IVP} \begin{cases} \dfrac{dy}{dx} = y^2 \\ y(1) = 2 \end{cases}$$

a) Show $y(x) = \dfrac{1}{C-x}$ solves the DE

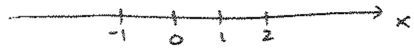
b) Solve the IVP.

Example 2 Consider motion along a straight line

position $x(t)$

velocity $x'(t) = v(t)$

acceleration $x''(t) = a(t)$



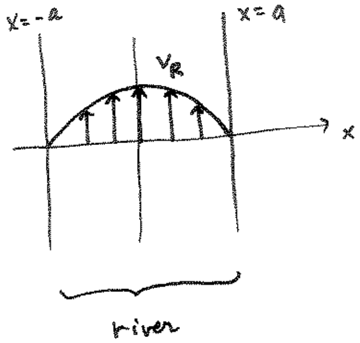
Suppose acceleration is constant, $x'' = a$ a constant

a) Solve for position and velocity, writing $x(0) = x_0$, $x'(0) = v_0$

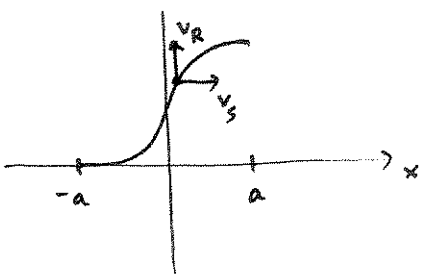
b) Solve this application:

A driver involved in an accident claims he was going only 25 mph. When police tested his car, they found that when its brakes were applied at 25 mph the car skidded only 45 ft before coming to a stop. But the driver's skid marks at the accident scene measured 210 ft. Assuming the same (constant) deceleration, determine the speed he was actually traveling just prior to the accident.

example 3 (actually example 4, the swimmer's problem, page 16)



$v_R = v_0\left(1-\frac{x^2}{a^2}\right)$ is velocity profile of river water



swimmer swims due east with constant velocity v_s.
she starts at $(-a,0)$.

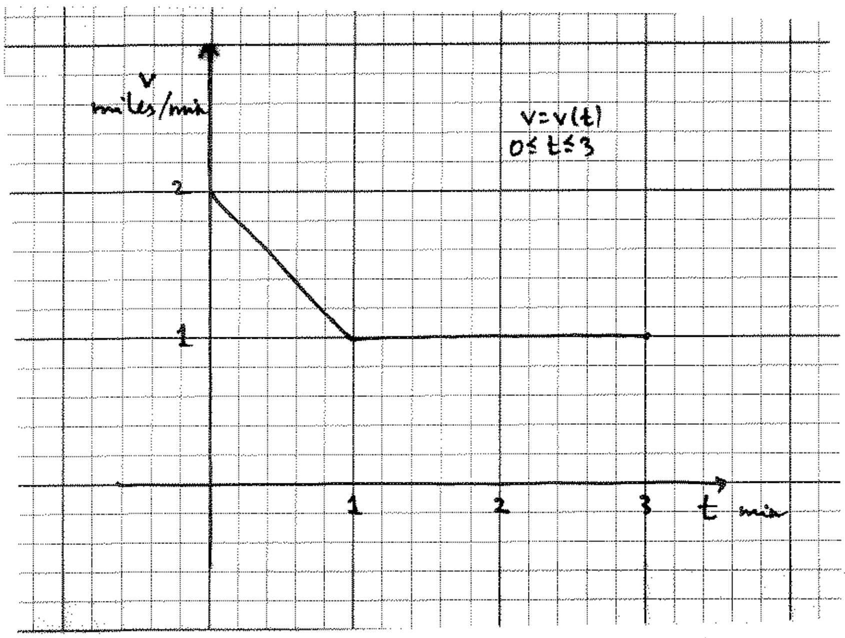
find the function $y=f(x)$ whose graph is describing her route!!

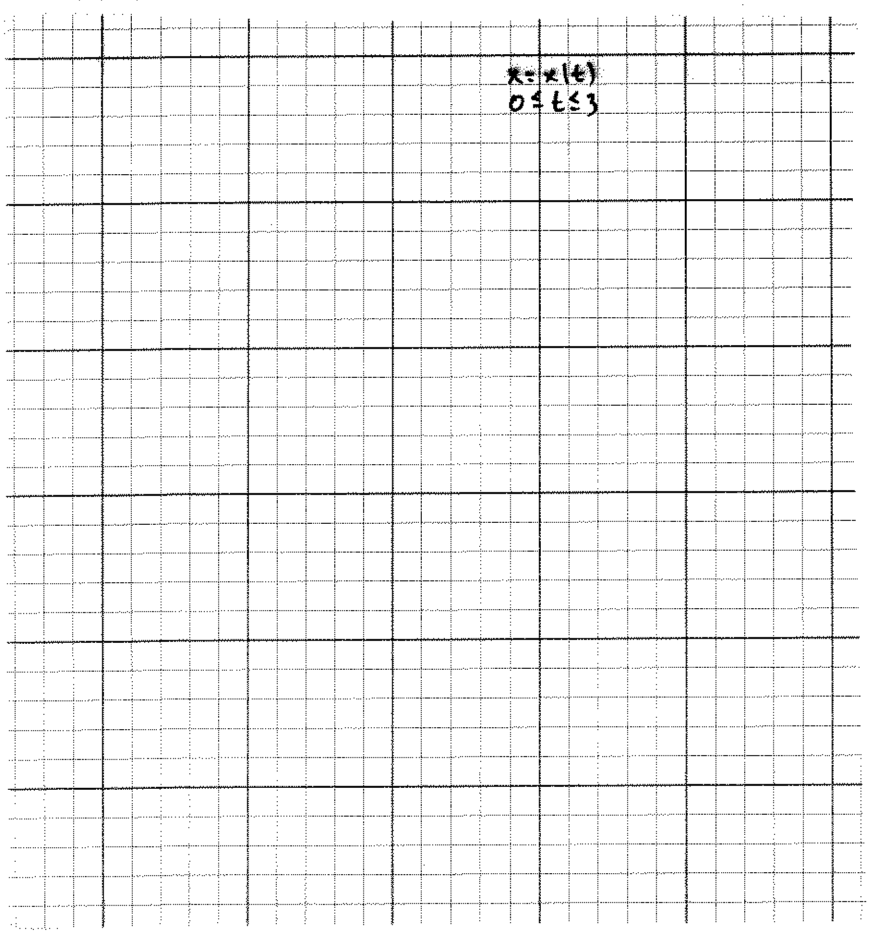
then, if ~~t~~ river is 1 mile wide
if $v_s = 3$ mi/h (this is a very good swimmer)
$v_0 = 9$ mi/h (pretty fast river)

where does swimmer land on opposite shore?

ans: $y=2$.

example 4 The velocity $v = \frac{dx}{dt}$ of a car traveling along the x-axis is shown in the following graph. The car starts at $x_0 = 0$.
Sketch the graph of its location at time t (and, find a formula for $x(t)$!)
How far did the car travel, $0 \le t \le 3$ minutes?





4b) extra: could you find exact formula for $x(t)$ i.e.

$$x(t) = \begin{cases} ? & 0 \le t \le 1 \\ ?? & t \ge 1 \end{cases}$$