Example 2 aug23.pdf

straight line motion with prescribed constant acceleration

$\boxed{x''(t) = a}$ (const)

$\Rightarrow$ $x'(t) = \int a\,dt = at + C$

@ $t=0$: $x'(0) = v_0 = C$, so

$\boxed{x'(t) = at + v_0}$

$\Rightarrow$ $x(t) = \int at + v_0 = \frac{1}{2}at^2 + v_0 t + C$

@ $t=0$: $x(0) = x_0 = 0 + 0 + C \Rightarrow C = x_0$

so $\boxed{x(t) = \frac{1}{2}at^2 + v_0 t + C}$

In the case of deceleration (negative acceleration) it may be more convenient to write the acceleration as $-a$, with $a>0$.

$$\boxed{\begin{aligned} x''(t) &= -a \quad \text{with } a>0 \\ \Rightarrow x'(t) &= -at + v_0 \\ x(t) &= -\frac{1}{2}at^2 + v_0 t + x_0 \end{aligned}}$$

Steps towards solving the accident problem:

- for given v_0, how far until stop?
  at stopping time t_s, $x'(t_s) = 0 = -at_s + v_0$
  so $\boxed{t_s = \frac{v_0}{a}}$

  set $x_0 = 0$, the distance traveled is

$$x(t_s) = -\frac{1}{2}at_s^2 + v_0 t_s$$

$$x(t_s) = -\frac{1}{2}a\left(\frac{v_0}{a}\right)^2 + v_0\left(\frac{v_0}{a}\right) = \boxed{\frac{1}{2}\frac{v_0^2}{a}}$$

2b) Car accident. Driver skids 210 ft.
Claims speed before skid was 25 miles/h
Police test: 25 mph ⇒ skid of 45 ft.

Assuming constant deceleration, what was actual speed before braking into skid?

letters:

v_P: initial speed in police test., D_P: distance of skid in Police test
v_A: initial speed in accident. D_A: actual skid distance in accident.

from page 1,

$$D_P = \frac{1}{2}\frac{v_P^2}{a}, \quad \text{so } a = \frac{v_P^2}{2D_P} \quad \text{(can be determined from given data)}$$

therefore, since also

$$D_A = \frac{1}{2}\frac{v_A^2}{a},$$

$$v_A^2 = 2aD_A$$

$$v_A^2 = 2\left(\frac{v_P^2}{2D_P}\right)D_A = v_P^2\frac{D_A}{D_P}$$

so $v_A = v_P\sqrt{\frac{D_A}{D_P}}$

since $\frac{D_A}{D_P}$ is independent of length units (as long as each is measured in the same units),

we get $v_A = 25\sqrt{\frac{210}{45}}$ miles/hour

$$\boxed{v_A \cong 54 \text{ miles/hour}}$$

the discussion of unit conversion, miles/hour → ft/sec was useful in class Tuesday, but by working symbolically for as long as possible we are actually able to avoid those details

1 mile/hour

$$= 1\,\frac{\text{mile}}{\text{hour}} \cdot \frac{5280\text{ ft}}{1\text{ mile}} \cdot \frac{1\text{ hour}}{3600\text{ sec}}$$

$\cong 1.467$ ft/sec