Math 2250-1
(& 2250-6)

Monday August 22

- Course information on syllabus & homepage

Homework due 8/25 (<u>Thursday</u>)

1.1 5, 6, 8, 9, 15, 16, 19, 29, 32-36, 45, 46

1.2 6, 7, 10, 18, 20, 25, 33, 34, 40

Circled problems are to be handed in.

- what is a differential equation?

  - n^{th} order DE : $F(x, y, y', y'', \ldots y^{(n)}) = 0$

    i.e. an equation involving a variable "x" and an (unknown) function y(x) & its first n derivatives

- where do differential eqtns come from?

  - mathematical models, often of continuous dynamical systems in which the variable "x" is actually time "t".

- goal:

  - understand the "solution function(s)" y(x), i.e. the functions for which the differential equation is true.

    (If you are able to find all the solutions this is called <u>solving</u> the D.E.)

Examples you've probably seen in Calculus and physics:

1^{st} order differential equations : rate of change of a function depends in some way on the function value, the variable value, and nothing else.

e.g. population growth models

2^{nd} order differential equations : Newton's 2^{nd} law often leads to such DE's.

mass · acceleration equals net forces

Let's recall some of these, and how we solve them.

1 Population growth or decay model.

applications?

$$\frac{dP}{dt} = kP$$

"x" = t (time)

"$y(x)$" = $P(t)$ (population or amount)

model:
rate of change of population is proportional to the population

Solve this DE!

Chain rule backwards

same as using differentials (for separable DE's)

2 Newton's Law of Cooling

A = ambient temperature

solid object with temp. $T(t)$

model: rate of change of temperature is proportional to difference between temperature and ambient temperature

yields DE:

$$\frac{dT}{dt} = -k(T-A)$$

why did I write $-k$?
would DE be correct if I just wrote k?

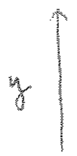
Use this model to solve a murder mystery:

At 3:00 a.m. a human body is discovered, with body temp. = 85°
by 4:00 a.m. body temp = 80°. Ambient temperature remains constant 65°

About when did murder occur?

3 motion with prescribed acceleration via Newton's 2^{nd} law.

e.g.



$m y''(t) = -mg$

$y(t)$ = height at time t

g = acceleration of gravity $\quad 9.8\ m/s^2$

$$y'' = -g \qquad \text{solve it!}$$

$$y' =$$

4 conceptual example

a) Show $y(x) := \frac{1}{C-x}$ solves $\frac{dy}{dx} = y^2$, where C is any constant.

b) solve the initial value problem

$$\begin{cases} \frac{dy}{dx} = y^2 \\ y(1) = 2 \end{cases}$$