Math 2250
Friday 9/3

I will be in the Math Dept computer classroom $\boxed{\text{LCB 115,}}$ tomorrow (Saturday), $\boxed{\text{11:00–1:00.}}$ (Normally all Ugrad facilities closed on weekend, in Math Dept.)

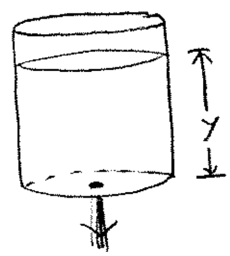
- I will do a Maple tutorial for beginners
- Anyone can work on project I, and I can answer questions

Tutorial & project are posted at our home page
www.math.utah.edu/~korevaar/2250fall04

Torricelli Experiment! : Model on page 1 Wed, for general cistern

Model: cylindrical container

$$\begin{cases} \frac{dy}{dt} = -k y^{1/2}, \quad \text{since } A(y) \text{ is constant} \\ y(0) = y_0 \end{cases}$$



Find when tank empties, given that you measure $y(t_1) = y_1$

$$y^{-1/2}\, dy = -k\, dt$$

integrate:

$$2y^{1/2} = -kt + C$$

$$y^{1/2} = -\frac{k}{2}t + \tilde{C}$$

$y(0) = y_0$ so $\tilde{C} = \sqrt{y_0}$

(1) $$\boxed{\sqrt{y} = -\frac{k}{2}t + \sqrt{y_0}}$$

use $y(t_1) = y_1$ to find k:

$$\sqrt{y_1} = -\frac{k}{2}t_1 + \sqrt{y_0}$$

$$k\frac{t_1}{2} = \sqrt{y_0} - \sqrt{y_1}$$

(2) $$\boxed{k = \frac{2(\sqrt{y_0} - \sqrt{y_1})}{t_1}}$$

and then deduce when $y = 0$ from (1), (2):

$$0 = -\frac{k}{2}t + \sqrt{y_0}$$

$$t = \frac{2\sqrt{y_0}}{k} = \frac{t_1\sqrt{y_0}}{\sqrt{y_0} - \sqrt{y_1}}$$

(3) $$\boxed{t = \frac{t_1\sqrt{y_0}}{\sqrt{y_0} - \sqrt{y_1}}}$$

reality: Nalgene liter canteen

$y_0 = 15.9$ cm (V = 1000 ml)
$y_1 = 12.65$ cm (V = 800 ml

(a) Find $t_1 = \boxed{}$ (experimentally) ← I got 112.9 sec at home

(b) so, from our model (3)

$$t = \frac{t_1\sqrt{15.9}}{\sqrt{15.9} - \sqrt{12.65}}$$

predict $t = \boxed{}$ ← with my t_1, I predict 121 sec

(c) and, when we run the experiment we get

$t = \boxed{}$ ← I actually measured 117 sec.

§1.5 Linear 1st order DE's

$$y' + P(x)y = Q(x)$$

sol'n method:

any antideriv of P(x) $\longrightarrow$

$$\underbrace{e^{\int P(x)dx}\left[y' + P(x)y\right]} = e^{\int P(x)dx} Q(x)$$

by product rule (& chain rule) $\longrightarrow$

$$= \left[e^{\int P(x)dx} y\right]'$$

so can antidifferentiate both sides (wrt x)

$$e^{\int P(x)dx} y = \int e^{\int P(x)dx} Q(x)\,dx + C$$

$$\boxed{y = Ce^{-\int P(x)dx} + e^{-\int P(x)dx}\int e^{\int P(x)dx} Q(x)\,dx}$$

Memorize the method, <u>NOT</u> the solution formula!

<u>example 2</u>, p. 48-49

$$(x^2+1)y' + 3xy = 6x$$

$$y' + \frac{3x}{x^2+1}y = \frac{6x}{x^2+1}$$

$P = \frac{3x}{x^2+1}$

$\int P\,dx = \int \frac{3x}{x^2+1}dx = \frac{3}{2}\ln(x^2+1) \quad (+C)$

$e^{\int P dx} = e^{\frac{3}{2}\ln(x^2+1)} = (x^2+1)^{3/2}$!

$$\underbrace{e^{\int P(x)dx}\left[\quad\right]} = e^{\int P(x)dx}\frac{6x}{x^2+1}$$

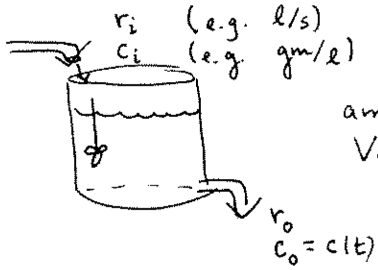
$$\left[e^{\int P(x)dx} y\right]'$$

$$\left[(x^2+1)^{3/2} y\right]' = (x^2+1)^{3/2}\frac{6x}{(x^2+1)} = 6x(x^2+1)^{1/2}$$

$$(x^2+1)^{3/2} y = \int 6x(x^2+1)^{1/2}dx = 2(x^2+1)^{3/2} + C$$

$$\boxed{y = 2 + C(x^2+1)^{-3/2}}$$

Mixing problems (p. 51–53)



amount $x(t)$ of solute (e.g. gm)

Vol $V(t)$

concentration $c = \dfrac{x(t)}{V(t)}$ (well-mixed hypothesis)

$$\left(\frac{gm}{s}\right) \qquad \frac{dx}{dt} = c_i r_i - c_o r_o = c_i r_i - r_o \frac{x(t)}{V(t)}$$

$$\frac{dV}{dt} = r_i - r_o$$ $\longleftarrow$ so if know V_o, $r_i(t)$, $r_o(t)$, can integrate to find $V(t)$, for $\frac{dx}{dt}$ eqtn

$$\boxed{\frac{dx}{dt} + \frac{r_o}{V} x = c_i r_i}$$ linear

example 4 p 52 (Lake Erie)

$V = 480 \text{ km}^3$

$r_i = r_o = 350 \text{ km}^3/\text{year}$

$c_i = c =$ pollutant concentration of Huron. (constant)

at $t = 0$, Erie concentration $= 5c$

so $x(0) = 5cV$

When is $x(t)$ only $2cV$?

$$\begin{cases} \dfrac{dx}{dt} = rc - r\dfrac{x(t)}{V} \\ x(0) = 5cV \end{cases}$$

$x'(t) + \frac{r}{V} x(t) = rc$ $\qquad$ $P(t) = \frac{r}{V}$ (const); $\quad e^{\int P dt} = e^{\frac{r}{V}t}$

$\left(e^{\frac{r}{V}t} x\right)' = e^{\frac{r}{V}t} rc$

$e^{\frac{r}{V}t} x = cVe^{\frac{r}{V}t} + C$

$x = cV + Ce^{-\frac{r}{V}t}$

$x(0) = 5cV$ so $C = 4cV$

$x(t) = cV + 4cVe^{-\frac{r}{V}t}$

Solve $2\cancel{cV} = \cancel{cV} + 4\cancel{cV}e^{-\frac{r}{V}t}$

$1 = 4e^{-\frac{r}{V}t}$

$\frac{1}{4} = e^{-\frac{r}{V}t}$

$-\ln 4 = -\frac{r}{V}t$

$t = \dfrac{V \ln 4}{r} = \dfrac{480 \ln 4}{350} \approx 1.9$ years!