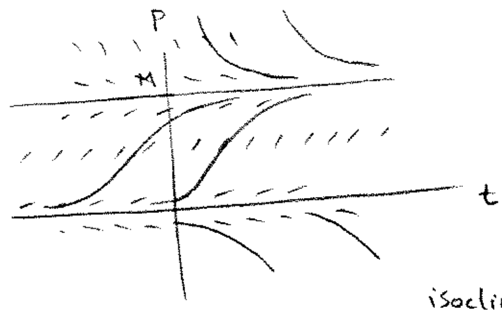
Math 2250-3
Friday 9/10

I (believe) I have reserved PC Lab 1735 in MMC labs (Marriott basement) 10–12 Saturday morning, if you want to work on Maple project with trouble shooter (me) present.

- Finish Logistic model of U.S. Population (Wed. notes)

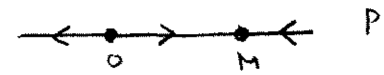
const solns $P=0, P=M$
these are called <u>equilibrium</u> <u>solutions</u>




isoclines are horizontal lines

sign $\frac{dP}{dt}$ : $- - - \ 0 \ + + + \ 0 \ - - -$ P ($0$, $M$)
$\quad\ ||$
sign $kP(M-P)$

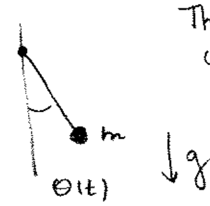
deduce that $P \nearrow$ or $\searrow$ as follows




we call $P=M$
a <u>stable equilibrium</u> because
if $P_0$ is near M then $P(t)$ stays near M for all $t>0$
(in fact M is <u>asymptotically</u> stable, i.e. if $P_0$ is near enough to M, then $\lim_{t\to\infty} P(t) = M$.)

we call $P=0$ an
<u>unstable</u> <u>equilibrium</u> because it is not true that if $P_0$ is near enough to 0 then $P(t)$ stays close to zero.

The notion of equilibrium solutions and stability is used throughout DE's – for example, consider a rigid rod pendulum, for which all $\theta$ are possible.
There are 2 equilibrium configurations, one is stable & one is unstable.



logistic :

fertility rate $\beta = \beta_0 - \beta_1 P$ , $\dfrac{\frac{dP}{dt}}{P} = \beta - \delta$ leads to $\frac{dP}{dt} = aP^2 - bP = kP(M-P)$.

morbidity $\delta = \delta_0 - \delta_1 P$

doomsday / extinction is a related population model, with wildly different dynamics :

Assume: an individual's probability of reproducing is proportional to the probability they encounter an individual of the opposite sex (think alligators).

Individuals cover territory of certain size, so prob of meeting partner is proportional to population density. If creatures live in fixed area pop density is prop to P

$\Leftarrow$ $\beta = kP$

slope field.

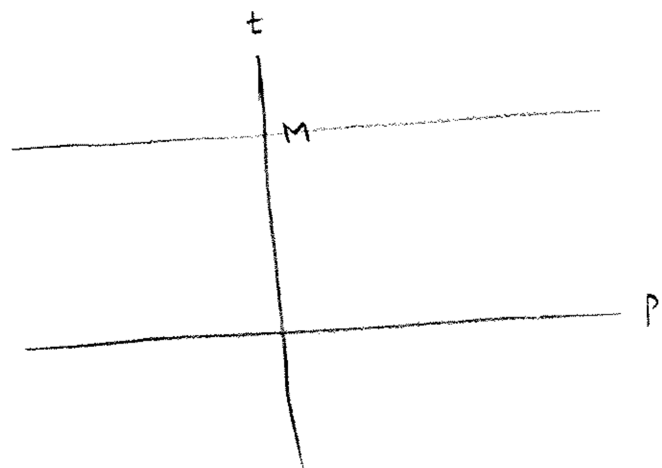
If $\beta = kP$ and $\delta = \delta_0$

get

$$\frac{\frac{dP}{dt}}{P} = kP - \delta_0$$

$$\frac{dP}{dt} = kP^2 - \delta_0 P$$

$$= kP(P-M)$$




sign $\frac{dP}{dt}$ : + + + 0 − − 0 + + +

0 M

dynamics → → → ← ← → → →

0 M

What are the equilibrium sol's?
Which are stable? unstable?

one of your HW problems (2.1 #33) is to solve this DE,

Find

$$P(t) = \frac{MP_0}{P_0 + (M - P_0)e^{Mkt}}$$

from which you can analytically verify what the slope field predicts.

General language & notions

If DE has form

$$\boxed{\frac{dx}{dt} = f(x)}$$

it is called <u>autonomous</u>

solutions which are constant in time are called $\boxed{\text{equilibrium solutions}}$

[ If $x(t) \equiv x_0$ is const soltn then

$\frac{dx}{dt} = 0$ and $\frac{dx}{dt} = f(x) = f(x_0)$ so $x_0$ is a root of $f(x)$,

and visa versa ]

<u>examples</u>

$\frac{dx}{dt} = kx(M-x)$ logistic

$\frac{dx}{dt} = kx(x-M)$ D/Ext

$\frac{dx}{dt} = k(A-x)$ Newton's Law of cooling

$\frac{dx}{dt} = x^3 - x$

$\frac{dx}{dt} = \sin x$

Let $c$ be an equilibrium soltn for $\frac{dx}{dt} = f(x)$

If, for each $\varepsilon > 0$ there exists a $\delta > 0$ s.t.

$|x_0 - c| < \delta$ implies $|x(t) - c| < \varepsilon$ for all $t > 0$

for sol'ns to IVP with $x(0) = x_0$

then $c$ is $\boxed{\text{stable equil.}}$ (solutions starting close to $c$ stay close to $c$)

<u>else</u> $c$ is $\boxed{\text{unstable equil.}}$

If $c$ is stable equil. & there exists $\delta > 0$ s.t.

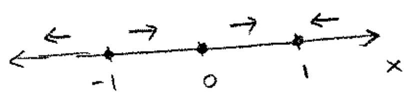
$|x_0 - c| < \delta$ implies $\lim_{t \to \infty} x(t) = c$ then

$c$ is $\boxed{\text{asymptotically stable}}$

<u>example</u> Find and classify equilibria for

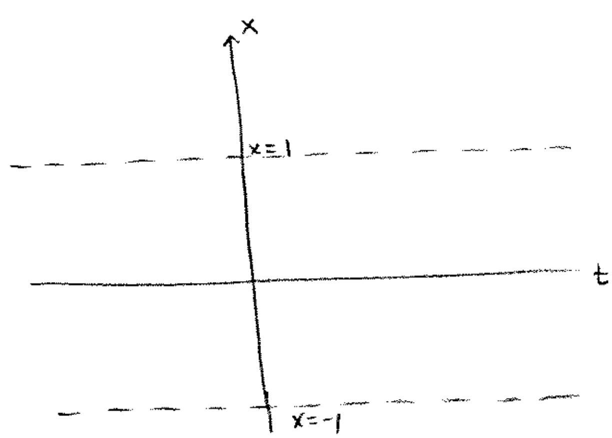
$$\frac{dx}{dt} = x^2 - x^4$$
$$= x^2(1-x^2)$$
$$= x^2(1-x)(1+x)$$

equil: $x = 0, 1, -1$




sign $\frac{dx}{dt}$    $- - - \; 0 \; + + + \; 0 \; + + + \; 0 \; - - -$

stability?




slope field and solution graphs?