Math 2250
Wed Sept 1

We'll do the MAPLE notes first, then discuss these two real examples. On Friday we'll do a Torricelli experiment.

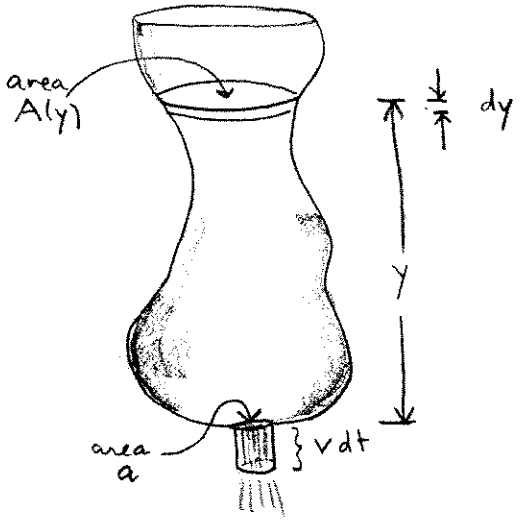
HW for Wed 9/8

1.4 3, 4, (9) (12), 14, 19, (20) (22) (40, 45, 49, 54, 61)
1.5 1, 7, (8) (13) (20) (33) 36, (38, 41)

Torricelli's Law for draining tanks:

the speed v with which water leaves hole is

$$\boxed{v = \sqrt{2gy}}$$



reason: KE + PE = const

in a small time interval dt a mass of water

$$dM = \rho\, dV = \rho\, A(y)\, dy$$

is lost from the top; replaced with equal mass

$$dM = \rho\, dV = \rho\, a\, v\, dt$$

shooting from bottom.

Since

loss in PE = gain in KE

$$(dM)\, g y = \tfrac{1}{2}(dM)\, v^2$$

$$v = \sqrt{2gy} \quad \blacksquare$$

We can express Torricelli as a separable DE by equating to two expressions for dM (or dV) on the right:

$$A(y)\, dy = a\, v\, dt$$

$$A(y)\, dy = a\sqrt{2gy}\, dt$$

$$\boxed{A(y)\frac{dy}{dt} = -k\sqrt{y}}$$

Newton's Law of Cooling.
(problem from 1st day of class).

Murder mystery

$65° = A$
3 a.m body temp = 85°
4 a.m " " = 80°

When did the body die, (using Newton's Law of Cooling model)?

$$\frac{dT}{dt} = k(A - T)$$

take $t = 0$ to be 3 a.m.

(I deduce death at $\approx$ 1:12 :)