Math 2250-3
Monday 10/18

§ 4.5 row and column spaces
(by the way, we will skip §4.6 and cover §4.7 on Wed.)

How's your vocab?

V vector space
W subspace

dimension for vector space (or subspace)
basis for vector space (or subspace).
numerology

On Friday we realized that our rref + backsolve algorithm always yields a basis for the solution space to

$$A\vec{x} = \vec{0}$$

(and it's dimension = # cols in rref(A) without leading 1's = # of free parameters).

Two other interesting subspaces associated to a matrix are

1. rowspace(A) = span of rows of A, a subspace of $\mathbb{R}^n$
2. colspace(A) = span of columns of A, a subspace of $\mathbb{R}^m$.

Playing with these subspaces gives more, concrete vector space examples.
We use two important ~~ex~~ tools:

1. Let $W = \text{span}\{\vec{v}_1, \ldots \vec{v}_k\}$.
   If you perform "elementary vector ops" to the v_j's i.e.
   1. interchange $\vec{v}_i$ & $\vec{v}_j$
   2. multiply $\vec{v}_j$ by a non-zero c
   3. replace $\vec{v}_j$ with $c\vec{v}_i + \vec{v}_j$

   to get a new set of vectors $\{\vec{w}_1, \ldots \vec{w}_k\}$
   you do not change the span.

   Can you see why?

2. if $\vec{v}_j$ is a linear combo of the remaining $\{\vec{v}_k\}$, then you may remove it from the set without shrinking the span.

   Can you see why?

Thus, we can get a nice basis for rowspace(A) by computing rref(A)!

→ elementary row ops don't change span of the rows, by 1,
so the rows of rref(A) are a basis for rowspace.

In example below

1. Find nice basis for row space
2. Find basis for soltn space $A\vec{x}=\vec{0}$
3. Remove dependent vectors from column vectors to get basis for column space *** ⇐ NEAT.
4. (alternately), compute & use rcef(A) to get nice basis for column space.
5. Explain why rowspace & colspace have same dimension (always!), (called "rank" of matrix) and why this dimension + the solution space dimension (to $A\vec{x}=\vec{0}$) add up to the # of columns of A. (always!).

Math 2250-3
Row and Column spaces of a Matrix
October 18, 2004

The rowspace of an m by n matrix A is the subspace of R^n spanned by the rows in A. The column space is the subspace of R^m spanned of the columns. The nullspace is the solution set to the homogeneous matrix equation Ax=0. We can use rref to figure out bases for the row and column spaces - we already saw how to use it to find bases for the nullspace. And it's kind of cute. We can also deduce the interesting fact that the row space and the column space have the same dimension (called the rank of the matrix), and that the rank plus the dimension of the nullspace add up to the number of columns "n". Here's an example we will discuss:

```
> with(linalg):
> A:=matrix(4,5,[1,2,1,3,2,3,4,9,0,7,2,3,5,1,8,
  2,2,8,-3,5]);
```

$$A := \begin{bmatrix} 1 & 2 & 1 & 3 & 2 \\ 3 & 4 & 9 & 0 & 7 \\ 2 & 3 & 5 & 1 & 8 \\ 2 & 2 & 8 & -3 & 5 \end{bmatrix}$$

```
> rref(A);
```

$$\begin{bmatrix} 1 & 0 & 7 & 0 & -39 \\ 0 & 1 & -3 & 0 & 31 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> transpose(rref(transpose(A)));
  #reduced column echelon form!
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

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