Math 2250-3
Wed 10/13

HW for 10/20
4.2 2, 6, 9, 27
4.3 16, 17, 18, 23, 25
4.4 1, 2, 3, 6, 8, 9, 11, 13, 26
4.5 1, 12, 16, 26, 27, 28, 17

Use technology to compute rref, if you want to save time!

Recall our new terminology:

<u>linear combination</u> of $\{\vec{v}_1, \ldots \vec{v}_k\}$
<u>span</u> $\{\vec{v}_1, \ldots \vec{v}_k\}$
$\{\vec{v}_1, \ldots \vec{v}_k\}$ <u>linearly independent</u>
" " <u>dependent</u>

<u>V a vector space</u>
examples.
W a <u>subspace</u> of the vectorspace V

<u>example1</u>: consider $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \text{ s.t. } x + 2y - 3z = 0 \right\}$ — you're used to saying "the plane $x+2y-3z=0$"

- show W is a subspace (by checking ($\alpha$), ($\beta$).

($\alpha$) Let $\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \in W$. Thus $x_1 + 2y_1 - 3z_1 = 0$
$x_2 + 2y_2 - 3z_2 = 0$

so $(x_1 + x_2) + 2(y_1 + y_2) - 3(z_1 + z_2) = 0$

so $\begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{bmatrix} \in W$

($\beta$) If $\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \in W$ then $x_1 + 2y_1 - 3z_1 = 0$

so $cx_1 + 2cy_1 - 3cz_1 = 0$

so $c\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \in W$

- Another way to see W is a subspace:

$x + 2y - 3z = 0$

rref: $\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \end{array}\right]$

$z = t$
$y = s$
$x = 3t - 2s$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

so $W = \text{span}\left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

<u>example2</u>: Is the plane $x + 2y - 3z = 1$ a subspace?

* Finish page 4 Monday, about the two most important ways of characterizing subspaces.
In the example 1 we characterized the plane both ways!

<u>example 3</u> $\text{span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \right\}$ must be a plane thru $\vec{0}$, in $\mathbb{R}^3$

Find the equation $ax + by + cz = 0$ for this plane

<u>Ans</u>: If $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in W$ then we can solve

$$c_1 \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 + 3c_2 \\ -c_1 \\ 2c_1 + c_2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{for } c_1, c_2$$

$$\begin{array}{r} \\ \\ \\ \end{array}\left[\begin{array}{cc|c} 1 & 3 & x \\ -1 & 0 & y \\ 2 & 1 & z \end{array}\right]$$

$$\begin{array}{r} -R_1 \\ R_2 \\ \\ \end{array}\left[\begin{array}{cc|c} 1 & 0 & -y \\ 1 & 3 & x \\ 2 & 1 & z \end{array}\right]$$

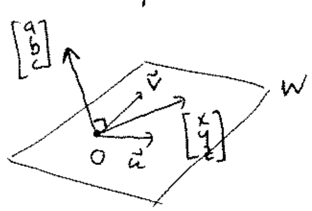
$$\begin{array}{r} \\ -R_1+R_2 \\ -2R_1+R_3 \end{array}\left[\begin{array}{cc|c} 1 & 0 & -y \\ 0 & 3 & x+y \\ 0 & 1 & z+2y \end{array}\right]$$

$$\begin{array}{r} \\ R_3 \\ R_2 \end{array}\left[\begin{array}{cc|c} 1 & 0 & -y \\ 0 & 1 & z+2y \\ 0 & 3 & x+y \end{array}\right]$$

$$\begin{array}{r} \\ \\ -3R_2+R_3 \end{array}\left[\begin{array}{cc|c} 1 & 0 & -y \\ 0 & 1 & z+2y \\ 0 & 0 & x+y-3(z+2y) = x-5y-3z \end{array}\right]$$

so $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in W$ iff $\boxed{x - 5y - 3z = 0}$

another way for this special case:




$ax + by + cz = 0$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & -1 & 2 \\ 3 & 0 & 1 \end{vmatrix} = (-1, 5, 3)$$

$-x + 5y + 3z = 0$ ✓

<u>example</u> Describe all subspaces of $\mathbb{R}^3$:

Let $W$ be such a subspace

(0) $\vec{0} \in W$.

that might be all!

if $W$ has more elements, let $\vec{u} \in W$, $\vec{u} \neq \vec{0}$

then $\mathrm{span}(\vec{u}) \subset W$

$\uparrow$ "is a subset of"

(1) that might be all. In this case $W = \{ t\vec{u},\ t \in \mathbb{R} \}$ is a <u>line thru $\vec{0}$</u>

if $W$ has more elts, let $\vec{v} \in W$, $\vec{v} \notin \mathrm{span}\,\vec{u}$.

then $\{\vec{u}, \vec{v}\}$ are linearly independent

and $\mathrm{span}\{\vec{u}, \vec{v}\} \subset W$.

(2) that might be all. In this case $W = \{ t\vec{u} + s\vec{v},\ s,t \in \mathbb{R} \}$

is a plane.

See last example;

$$\left[\begin{array}{c|c|c} \vec{u} & \vec{v} & \begin{matrix} x \\ y \\ z \end{matrix} \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & \sim \\ 0 & 1 & \sim \\ 0 & 0 & ax+by+cz \end{array}\right]$$

if that's not all, then let $\vec{w} \in W$, $\vec{w} \notin \mathrm{span}\{\vec{u}, \vec{v}\}$.

Then
$$\left[\begin{array}{c|c|c} u & v & w \end{array}\right] \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(otherwise ~~either $\vec{v}$ and $\vec{u}$ are dependent or~~ $\vec{w}$ is a l.c. of $\vec{u}, \vec{v}$),

Thus $\mathrm{span}\{\vec{u}, \vec{v}, \vec{w}\} = \mathbb{R}^3$

(3). So $W = \mathbb{R}^3$

<u>Def</u> A <u>basis</u> for the vector space $V$ is a linearly independent set $\{\vec{v}_1, \ldots \vec{v}_k\}$ in $V$ which also spans $V$

<u>Def</u> The number of elements in a (any!) basis for $V$ is the <u>dimension</u> of $V$.

We exhibited the dimension 0, 1, 2, 3 subspaces of $\mathbb{R}^3$.