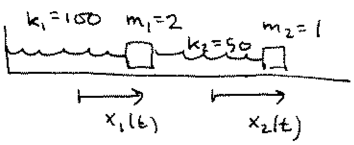
Math 2250-3
Wednesday 11/24

HW for Wed Dec 1
7.3 3, 4, 6, 13, 14, 25, 30, 32, 37 ← (postponed from this week)
7.4 2 3 8 13 14 16, 17, 18

• Finish Glucose-Insulin model from Monday's notes. Then do springs:

Spring systems §7.4 (We will finish this section Monday)

Example 1 p 431.



$$2x_1'' = -100x_1 + 50(x_2 - x_1)$$

$$1x_2'' = -50(x_2 - x_1)$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \begin{bmatrix} -150 & 50 \\ 50 & -50 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$M\vec{x}'' = K\vec{x}$$

in general, with n masses and up to $n+1$ springs, M & K are $n\times n$ matrices

What is the dimension of the solution space? (Careful!) ←

$\Rightarrow$ $\vec{x}'' = A\vec{x}$ where $A = M^{-1}K$.

$$\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$ in this example.

Look for solutions

$\vec{x}(t) = \cos\omega t\,\vec{v}$ & $\sin\omega t\,\vec{v}$, based on previous experience and conservation of energy:
if $e^{\lambda t}\vec{v}$ is a solution $\operatorname{Re}\lambda$ must $=0$ or else solution will either decay or grow exponentially, violating constant total energy!

$x' = -\omega\sin\omega t\,\vec{v}$
$x'' = -\omega^2\cos\omega t\,\vec{v}$
$A\vec{x} = \cos\omega t\,A\vec{v}$

must have
$A\vec{v} = -\omega^2\vec{v}$
$\vec{v}$ an eigenvector of A with eval $\lambda = -\omega^2$.

• hope A has n lin ind evects.
Each evect gives 2 l.i. solns $\cos\omega t\,\vec{v}$, $\sin\omega t\,\vec{v}$
$\Rightarrow$ $2n$ lin ind. solns
$\Rightarrow$ basis.

So, find general soltn to

$$\begin{bmatrix} x_1'' \\ x_2'' \end{bmatrix} = \begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Do you get

$$\vec{x}(t) = \underbrace{\left(c_1 \cos 5t + c_2 \sin 5t\right)}^{C_1 \cos(5t-\alpha_1)} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \underbrace{\left(c_3 \cos 10t + c_4 \sin 10t\right)}^{C_2 \cos(10t-\alpha_2)} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Explain what this solution means physically.
If you force this spring system, what angular frequencies for the forcing cosine fcn would induce resonance?

Now do forced oscillations:

$$M\vec{x}'' = K\vec{x} + \vec{b}\cos\omega t$$

or $\boxed{\vec{x}'' = A\vec{x} + \vec{F}_0 \cos\omega t}$ $\qquad \vec{F}_0 = M^{-1}\vec{b}$.

try $\vec{x}_p = \vec{c}\cos\omega t$ [assuming ω is <u>not</u> one of the natural angular frequencies]

$$\vec{x}_p'' = \vec{c}(-\omega^2\cos\omega t)$$

$$\text{want } = A\vec{c}\cos\omega t + \vec{F}_0\cos\omega t$$
$$= (A\vec{c} + \vec{F}_0)\cos\omega t$$

$$-\omega^2\vec{c} = A\vec{c} + \vec{F}_0$$
$$-\vec{F}_0 = A\vec{c} + \omega^2 I\vec{c}$$
$$-\vec{F}_0 = (A + \omega^2 I)\vec{c}$$
$$\boxed{\vec{c} = (A + \omega^2 I)^{-1}(-\vec{F}_0)}$$

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trouble when $\omega^2 = -\lambda$
since $A - \lambda I$ is sing!

<u>example 3</u>

$$\vec{x}'' = \begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix}\vec{x} + \begin{bmatrix} 0 \\ 50 \end{bmatrix}\cos\omega t$$

$$\vec{x}_p = \vec{c}\cos\omega t$$

$$-\omega^2\vec{c}\cos\omega t = \begin{bmatrix} -75 & 25 \\ 50 & -50 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}\cos\omega t + \begin{bmatrix} 0 \\ 50 \end{bmatrix}\cos\omega t$$

$$\parallel$$

$$\begin{bmatrix} -\omega^2 c_1 \\ -\omega^2 c_2 \end{bmatrix}\cos\omega t$$

$$\begin{bmatrix} 0 \\ -50 \end{bmatrix} = \begin{bmatrix} \omega^2 - 75 & 25 \\ 50 & \omega^2 - 50 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$c_1 = \frac{\begin{vmatrix} 0 & 25 \\ -50 & \omega^2 - 50 \end{vmatrix}}{(\omega^2 - 75)(\omega^2 - 50) - 50 \cdot 25}$$

$$\boxed{c_1 = \frac{1250}{(\omega^2 - 25)(\omega^2 - 100)} \qquad c_2 = \frac{50(\omega^2 - 75)}{(\omega^2 - 25)(\omega^2 - 100)}}$$

Interpret!?