```
> restart:
```
Pendulum:
```
> l:=.01*(98.2-5.5);
  g:=9.78;
  omega[0]:=sqrt(g/l);
  T:=evalf(2*Pi/omega[0]);
  f:=evalf(omega[0]/(2*Pi));
```

$$l := 0.927$$

$$g := 9.78$$

$$\omega_0 := 3.248101262$$

$$T := 1.934417927$$

$$f := 0.5169513714$$

```
> 40.0/77;   #In my office 40 cycles were completed in 77 seconds.
             #We'll see how it goes in class!
```

$$0.5194805195$$

Mass-spring system:
```
> m:=.050;
  k:=solve(k1*(.01)*(44.3-36.25)=m*g,k1);
```

$$m := 0.050$$

$$k := 6.074534161$$

```
> omega[0]:=sqrt(k/m);
  T:=evalf(2*Pi/omega[0]);
  f:=evalf(omega[0]/(2*Pi));
```

$$\omega_0 := 11.02228122$$

$$T := 0.5700440030$$

$$f := 1.754250540$$

```
> evalf(50/30);   #I measured 50 cycles in 30 seconds,
    #so this is my experimental frequency:
```

$$1.666666667$$

```
> 1/%;   #experimental period
```

$$0.5999999999$$

```
> m1:=.050+(1/3*.020);
      #the ''effective mass'' of the springs is 1/3 their
      #actual mass, as far as kinetic energy is concerned.
  k:=solve(k1*(.01)*(44.3-36.25)=m*g,k1);
  omega[0]:=sqrt(k/m1);
  T:=evalf(2*Pi/omega[0]);
  f:=evalf(omega[0]/(2*Pi));
      #is this closer to what we get?
```

$$m1 := 0.05666666667$$

$$k := 6.074534161$$

$$\omega_0 := 10.35363036$$

$$T := 0.6068581829$$

$$f := 1.647831450$$

**Review Sheet**
Math 2250-3 November 10, 2004

Our exam covers chapters 4-5 of the text. Only scientific calculators will be allowed on the exam. But you can expect to be working with Maple output, in ways consistent with the practice exam below and the homework problems you have worked.

## Chapter 4:

At most 40% of the exam will deal directly with this material....but much of Chapter 5 uses these concepts, so beware.

**Know Definitions:**

(a) **Vector Space:** A collection of objects which can be added and scalar multiplied, so that the usual arithemetic properties (Page 240) hold. You do not need to memorize all eight of these properties. The key point is that not only is R^n a vector space, but also certain subsets of it are, and so are spaces made out of functions...because functions can be added and scalar multiplied (page 265.)

(b) **Subspace:** a subset of a vector space which is itself of vector space....to check whether a subset is actually a subspace you only have to show that sums and scalar multiples of subset elements are also in the subset (Theorem 1 page 242.) Examples of important subspaces are the set of homogeneous solutions to a matrix equation (which I called the nullspace of the matrix and which the book calls the solution space, page 243), the span of a collection of vectors (page 248), AND the set of homogenous solutions to a linear differential equation (section 5.2).

(c) A **linear combination** of a set of vectors {v1, v2, ...vn} is any expression $c1*v1 + c2*v2 + ... + cn*vn$. (page 246)

(d) The **span** of a set of vectors {v1, v2, ...vn} is the collection of all linear combinations. (page 248)

(e) A collection {v1, v2, ...vn} is **linearly dependent** if and only if some linear combinatation (with not all $c_i$'s = 0) adds up to the zero vector.

(f) A collection {v1, ... vn} is **linearly independent** if and only if the only linear combination of them which adds up to zero is the one in which all coefficients $c_i=0$. (page 249)

(g) A **basis** for a vector space (or subspace) is a set of vectors {v1, ..., vk} which space the space and which are linearly independent. (page 255.)

(h) The **dimension** of a vector space is the number of elements in any basis.

**Know Facts:**

(a) If the dimension of a vector space is n, then no collection of fewer than n vectors can span and every collection with more than n elements is dependent.

(b) n vectors in R^n are a basis if and only if the square matrix in which they are the columns is non-singular. So you can use det or rref as a test for basis in this case.

(c) Basically all linear independence and span questions in R^n can be answered using rref. (see below.)

(d) You can toss dependent vectors out of a collection without changing the span. In this manner you can take a spanning set and turn it into a basis.

**Do computations:**

(a) Be able to check whether vectors are independent or independent, e.g. problems page 248. (4.3) Know how to use rref to check for dependencies.

(b) Be able to find bases for the solution space to homogeneous equations, e.g. problems page 255 (4.4)

(c) Be able to find bases for rowspace and column space, e.g. problems page 263 (4.5)

## Chapter 5:

At least 60% of the exam will cover this material, and at least 30% of it will be from sections 5.4 and 5.5. (Answering questions from 5.4 and 5.5 almost always uses 5.1-5.3 material implicitly.)

### 5.1-5.3, 5.5  General theory:

**Linear differential equations** (page 296.)

**principle of superposition** (e.g. Theorem 1 page 296, also leads to the fact that the general solution y to the inhomogeneous equation is yp + yh, where yp is a particular solution, and yh is the general solution to the homogeneous equation. (Theorem 5 page 306.)  Also leads to a method for getting particular solutions which are sums of particular solutions for pieces of the right hand side.)

**homogeneous**  (L(y)=0).  Solution space is an n-dimensional vector space.  Know how to find it for constant coefficients, using exponentials and the resulting **characteristic equation** and **Euler formula** if necessary  (section 5.3 and problems).  What to do with repeated roots.  The Wronskian test for linear independence .

**nonhomogeneous**  (L(y)=f).  Know how to find particular solutions by the method of undetermined coefficients.  Variation of parameters will not be on the exam.  (Section 5.5 and problems)

**initial value problem, existence and uniqueness.**  Know how to solve initial value problems by finding yp, and yh, and then finding values of constants in yh to match initial conditions.

### 5.4 and 5.6: Mechanical vibrations and forced oscillations:

**unforced oscillations** (i.e. solutions to the homogeneous DE):

**undamped** (simple harmonic motion)

going from A*cos(wt) + B*cos(wt) to C*cos(wt-a).  (The ABC triangle, amplitude and phase.) derivation of spring equation from Newton's and Hooke's Laws.

**damped.**

**under-damped, over-damped, critically damped.**  Know how to recognize, and different forms of the             solution.

**forced oscillations:**

**undamped:**

**resonance,** and when it arises.  form of solution, as follows from general theory above.

**beating,** when w is close to w0.

**damped:**

general solution is sum of **steady state periodic**, with **transient**  How to find each piece, and express the

steady state periodic solution in amplitude- phase form.

**practical resonance,**  will occur if damping is small and driving frequency is near natural frequency.

Exam will be closed-book and closed note. Only scientific calculators will be allowed. There will 100 points possible, with point values indicated in the right margin. The problems below are intended to be "typical", however they are not intended to be exhaustive in their scope.....please consult the review sheet for the topics you are responsible for.

1) Consider the homogeneous differential equation

$$deqtn := \left( \frac{d^2}{dt^2} x(t) \right) + 8 \left( \frac{d}{dt} x(t) \right) + 20 \, x(t) = 0$$

1a) If this was modeling a mass-spring configuration like we studied in Chapter 5 of Edwards-Penney, and if the mass was 3 kg, what values of coefficient of friction and spring constant would lead to the differential equation above? (1 point for getting the units correct, 2 points for the correct numerical values).

(6 points)

1b) What kind of damping is exhibited by this mass-spring system?

(4 points)

1c) Find the general solution to this homogeneous differential equation

(5 points)

1d) Consider the same spring system, but now with a driving force F0(t)=9*cos(2t). Find the general solution to this inhomogeneous differential equation. Use the method of undetermined coefficients. Identify the steady periodic and transient pieces of the solution. Find the amplitude and phase of the steady periodic solution.

(20 points)

2) Here is a matrix:

$$A := \begin{bmatrix} 1 & 3 & -4 & -8 & 6 \\ 1 & 0 & 2 & 1 & 3 \\ 2 & 7 & -10 & -19 & 13 \end{bmatrix}$$

Here is its reduced row echelon form:
```
> rref(A);
```

$$\begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & -2 & -3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2a) Find a basis for the solution space (of homogeneous solutions) to Ax=0.

(10 points)

2b)  Explain what it means for a collection of vectors to be linearly dependent or linearly independent.

(5 points)

2c)  Are the first three columns of A linearly independent or linearly dependent?  If they are dependent, exhibit a dependency.  If they are independent, explain why.

(10 points)

2d)  Explain what it means for a collection of vectors to span a vector space.

(5 points)

2e)  Do the first three columns of A span all of R^3.  Explain your answer.

(5 points)

2f)  Find a basis for the rowspace of A.

(5 points)

3)  Consider the differential equation

$$deqtn := \left( \frac{d^3}{dx^3} y(x) \right) + 25 \left( \frac{d}{dx} y(x) \right) = 10$$

Find the solution to the initial value problem for this differential equation, with y(0)=4, D(y)(0)=0, D(D(y)(0))=10.

(25 points)