Math 2250-3
Monday Aug 3.

Problem sessions tomorrow for HW due Wed!

§1.3 Slope fields

$$\begin{cases} \dfrac{dy}{dx} = f(x,y) \\ y(x_0) = y_0 \end{cases}$$

geometric interpretation: the graph of the solution $y(x)$ has slope at any point $(x, y(x))$ on it, given by the formula $f(x,y)$.
Also, the graph passes through (x_0, y_0).

⟶ this can be interpreted with a "slope field"

Example 1 (from Friday)

IVP $$\begin{cases} \dfrac{dy}{dx} = x-3 \\ y(1) = 2 \end{cases}$$

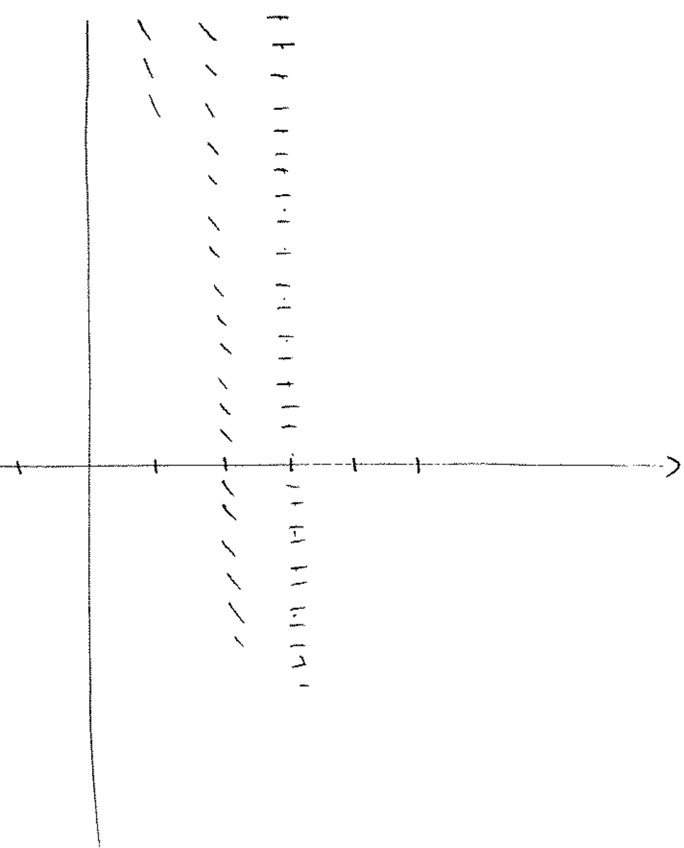
drawing by hand:

isoclines : curves along which the slope is constant

in this case, $x-3 = \text{const}$ ⟶ vertical lines

fill in!

curve	slope
$x=0$	-3
$x=1$	-2
$x=2$	-1
$x=3$	0
$x=4$	1
$x=5$	2



- fill in slope field picture
- ~~compare~~ draw the graph of soltn to IVP, using slope field
  compare to formula for soltn (from Friday)
- what do other solution graphs to same DE look like?

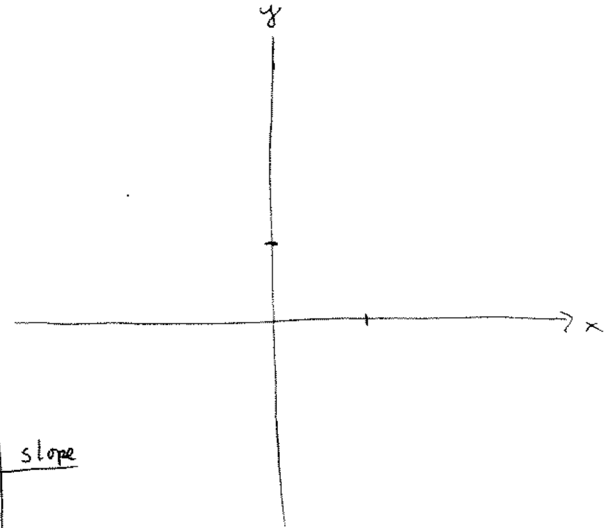
Example 2

Make the slope field for $\frac{dy}{dx} = y$

sketch the graph for the sol'n to

$$\begin{cases} \frac{dy}{dx} = y \\ y(0) = 1 \end{cases}$$

onto your slope field.
What is formula for soltn?
What do other solution graphs look like?



curve	slope

Example 3

(Is actually example 3 in new edition

$\frac{dv}{dt} = 32 - .16v$

down is positive direction

Discuss!

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> with(DEtools):
deqtn:=diff(v(t),t)=32-.16*v(t);  #this is example 3 page 21
dsolve({deqtn,v(0)=300},v(t));  #Maple solution for one IVP
DEplot(deqtn,v(t),t=0..25,{[v(0)=200],[v(0)=260],
   [v(0)=320],[v(0)=400],[v(0)=140],[v(0)=80],
   [v(0)=0]},v=0..400,
   arrows=line, color=black,linecolor=black,
   dirgrid=[30,30],
  title='Slope field for velocity of falling baseball');
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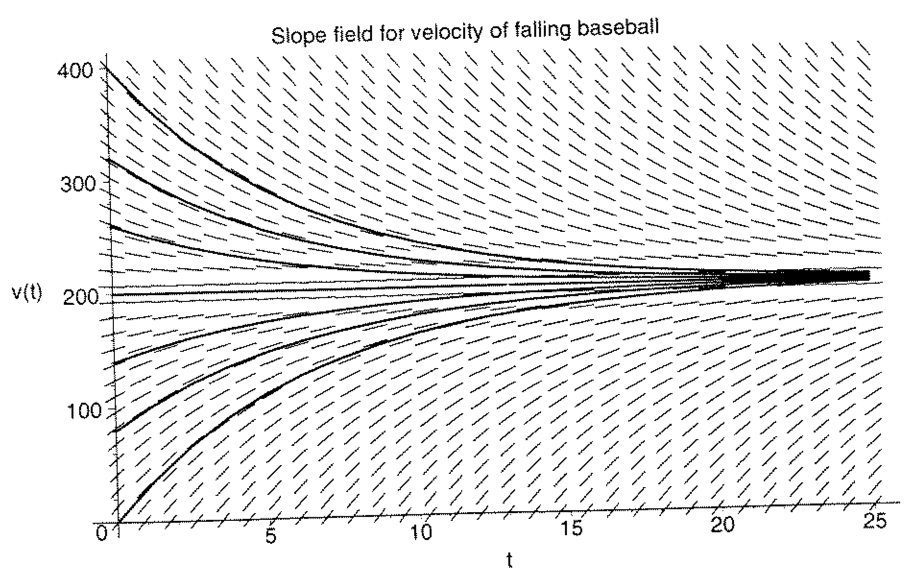
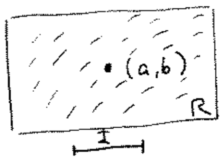
$$deqtn := \frac{d}{dt}\mathrm{v}(t) = 32 - 0.16\,\mathrm{v}(t)$$

$$\mathrm{v}(t) = 200 + 100\,\mathbf{e}^{\left(-\frac{4t}{25}\right)}$$



Existence and uniqueness Theorem page 23

Suppose $f(x,y)$ and its partial derivative $D_y f(x,y)$ are continuous on some rectangle R in the x-y plane that contains the point (a,b) in its interior



Then, for some open interval I containing a, the initial value problem

$$\begin{cases} \frac{dy}{dx} = f(x,y) \\ y(a) = b \end{cases}$$

has one, and exactly one solution defined on I

example 6 page 24

$$\frac{dy}{dx} = 2\frac{y}{x} \quad \text{or} \quad \boxed{x\frac{dy}{dx} = 2y}$$

$$\frac{dy}{y} = \frac{2}{x}\,dx$$

$$\ln|y| = 2\ln|x| + C$$

$$e \qquad e$$

$$|y| = e^C |x|^2$$

$\to$ $y \equiv 0$

$y = Cx^2$ solutions.

Consider

$$\begin{cases} \frac{dy}{dx} = \frac{2y}{x} \\ y(0) = 0 \end{cases}$$

$$\begin{cases} \frac{dy}{dx} = \frac{2y}{x} \\ y(1) = 1 \end{cases}$$