Math 2250-3
Wed 25 Aug

§1.1 Introduction to differential equations

HW due 9/1 (circled means hand in)

1.1	3, 4, 7, 15, 16, 19, 20, 27, 30, 34, 36, 46
1.2	5, 7, 10, 14, 20, 25, 26, 36, 49
1.3	3, 6, 8, 11, 12, 14, 21, 25, 32, 33

- A differential equation is an equation

$$F(x, y, y', y'', \dots y^{(n)}) = 0$$

where x is a name for the variable (domain an interval of $\mathbb{R}$)
(sometimes use "t" for this variable)
and $y(x)$ is a fcn, with derivatives $y', y'', \dots y^{(n)}$
the order of the DE is the highest order deriv which appears ("n")

- goal: Find the functions $y(x)$ which make the DE a true equality.
(This is called solving the DE).
Perhaps $y(x)$ will be required to satisfy further conditions as well, e.g. initial conditions.

- where do DE's come from?
  - They are often the result of mathematical models in science, engineering, everywhere!

Examples you have seen

(1) $\frac{dP}{dt} = kP$

k constant

model: "rate of change of $P(t)$ is proportional to $P(t)$"

$k > 0$ simple population growth
$k < 0$ radioactive (or other) decay

how you solved this DE:

Chain rule backwords:	Differentials:

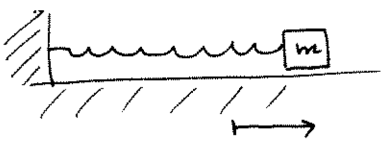
(2) Newton's Law of cooling (or heating)

$$\frac{dT}{dt} = k(A-T)$$

$T(t)$ = temperature at time t of object
A = constant ambient temperature

"how fast temp. changes is proportional to difference between $T(t)$ and A"

(3) springs:



$x(t)$
displacement from equilibrium

Newton's Law

$$mx'' = -kx - \mu x'$$

Hooke's constant

coeff. of friction.

if $\mu = 0$ get simple harmonic motion; trig fcn solns.

(4) projectiles:

$$my''(t) = -mg$$

$y(t)$ = height at time t
g = accel. of gravity = $9.8\ m/sec^2$

y

Solve it!

$y'' = -g$
$y' =$

conceptual
Example: (a) Show $y(x) = \frac{1}{C-x}$ solves $\frac{dy}{dx} = y^2$

(b) solve the initial value problem

$$\begin{cases} \frac{dy}{dx} = y^2 \\ y(1) = 2 \end{cases}$$

"real" example

Murder mystery

$65° = A$

3 a.m. body temp $85°$

4 a.m. " " $80°$

When did body die, estimate with Newton's Law of cooling.

$$\frac{dT}{dt} = k(A-T)$$

$$\frac{dT}{A-T} = k\,dt$$